# IWMS'2016

## Madeira Portugal

**June, 6 - 9**

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# BOOK OF ABSTRACTS

30 anos Ao serviço da Região — ipt Instituto Politécnico de Tomar

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IWMS'2016
Madeira Portugal

# The 25$^{\text{th}}$ International Workshop on Matrices and Statistics

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# Book of Abstracts

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**June 6 – 9, 2016**
**Madeira, Portugal**

***Edited by***

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# Contents

## Part V. Abstracts

# Part I

# Introduction

The 25$^{\text{th}}$ International Workshop on Matrices and Statistics (IWMS'2016) will be held on June 6-9, 2016 in the city of Funchal at the beautiful Madeira Island, the Pearl of the Atlantic.

The purpose of the workshop is to bring together researchers sharing an interest in a variety of aspects of statistics and its applications as well as matrix analysis and its applications to statistics, and offer them a possibility to discuss current developments in these subjects. The workshop will bridge the gap among statisticians, computer scientists and mathematicians in understanding each other's tools. We anticipate that the workshop will stimulate research, in an informal setting, and foster the interaction of researchers in the interface between matrix theory and statistics.

Some emphasis will be put on related numerical linear algebra issues and numerical solution methods, relevant to problems arising in statistics.

The workshop will include invited talks given by

- Alan Agresti (USA)
- Rosemary A. Bailey (UK)
- Radosław Kala (Poland)
- Alexander Kovačec (Portugal)
- Jianxin Pan (UK)

as well as two special sessions:

- **Session devoted to the 75$^{\text{th}}$ birthday of Professor Jeffrey J. Hunter**
  Chair of this session is Peter G. Taylor (Australia) with invited talks from:
  - Jeffrey J. Hunter (New Zealand)
  - Tuğrul Dayar (Turkey)
  - Steve Kirkland (Canada)
  - Guy Latouche (Belgium)
  - Peter G. Taylor (Australia)

- **The Memorial Session of Ingram Olkin**
  Chair of this session is Hans Joachim Werner (Germany) with invited talks from:
  - Hans Joachim Werner (Germany)
  - Jeffrey J. Hunter (New Zealand)
  - Simo Puntanen (Finland)
  - Michael Greenacre (Spain): video

The following minisymposia devoted to cutting edge research topics will be held during the workshop:

- Magic Squares, Prime Numbers and Postage Stamps
  organized by Ka Lok Chu (Canada)
- Multivariate Linear Models
  organized by Katarzyna Filipak (Poland)
- Inequalities in Matrix Theory and Probability
  organized by Alexander Kovačec(Portugal)
- Methods for Modelling Correlated and Complex Data
  organized by Jianxin Pan (UK)
- Estimation and Testing in Linear Models
  organized by Roman Zmyślony (Poland)

## Committees and Organizers of IWMS'2016

### The International Scientific Committee

- Simo Puntanen (Finland) - Chair
- George P.H. Styan (Canada) - Honorary Chair
- Júlia Volaufová (USA) - Vice-Chair
- S. Ejaz Ahmed (Canada)
- Katarzyna Filipiak (Poland)
- Jeffrey J. Hunter (New Zealand)
- Augustyn Markiewicz (Poland)
- Dietrich von Rosen (Sweden)
- Hans Joachim Werner (Germany)

### The Organizing Committee

- Francisco Carvalho (Portugal) - Chair
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- Ana Maria Abreu (Portugal)
- Daniel Klein (Slovakia)

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- CMA - Centro de Matemática e Aplicações (FCT, UNL)
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- PSE - Produtos e Serviços de Estatística
- INE - Instituto Nacional de Estatística
- FLAD - Fundação Luso-Americana para o Desenvolvimento
- Delta Cafés
- Associação de Promoção da Madeira

# Part II

# Program

# Program

## Sunday, June 5, 2016

**16:30 – 18:00 Welcome Drink at the Instituto do Vinho da Madeira**

## Monday, June 6, 2016

**9:00 – 9:20 Registration**

**Plenary Session**

9:20 – 9:30 *Opening*
9:30 – 10:10 J. Pan: *Joint mean-covariance modelling and its R package: jmcm*

**Contributed Session I**

10:20 – 10:40 P. Song: *Method of divide-and-combine in regularized regression for Big Data*
10:40 – 11:00 H. Drygas: *Adding observations in regression analysis*
11:00 – 11:20 K. Vaňkátová: *Clusterwise regression using mixtures of regression models*

**11:20 – 11:40 Coffee Break**

**Minisymposium** - Multivariate Linear Models

11:40 – 12:00 D. Klein: *Testing mean under compound symmetry covariance setup*
12:00 – 12:20 I. Žežula: *Comparison of estimators in a multivariate linear model with generalized uniform correlation structure*
12:20 – 12:40 M. Fonseca: *Estimation for the growth curve model with orthogonal covariance structure*

**12:40 – 14:10 Lunch**

**Contributed Session II**

14:10 – 14:30 A. Markiewicz: *Optimal circular neighbor designs under mixed interference models*
14:30 – 14:50 C. Francisco: *Hadamard matrices on error detection and correction: Useful links to BIBD*

**Minisymposium** - Estimation and Testing in Linear Models

15:00 – 15:20 J. T. Mexia: *Normal approximations to noncentral Wishart matrices*
15:20 – 15:40 J. S. Allison: *Goodness-of-fit tests for semiparametric transformation models*
15:40 – 16:00 L. Santana: *Goodness-of-fit tests for semiparametric and parametric hypotheses based on the probability weighted empirical characteristic function*

**16:00 – 16:20 Coffee break**

**Minisymposium** - Magic Squares, Prime Numbers and Postage Stamps

16:20 – 16:40 K. L. Chu: *Se-tenant philatelic Machin-definitive blocks with selected total face-values, with special emphasis on the Royal Mail "Stamps for Cooks" Prestige Booklet*
16:40 – 17:00 G. P. H. Styan: *A sensational* $7 \times 7$ *pandiagonal magic square with non-consecutive entries and "diamond-square arrangement" matrices for knight's tours in a pandiagonal magic carpet*
17:00 – 17:20 K. L. Chu: *Some comments on Margaret Kepner's "Magic Square 25 Study" (2010)*
17:20 – 17:40 G. P. H. Styan: *Some comments on Sophie Germain prime numbers and on two philatelic magic-carpet dinner-placemats for the IWMS-2016 Madeira Magic Minisymposium*

## Tuesday, June 7, 2016

**Plenary Session**

9:30 – 10:10 A. Kovačec: *The 123 theorem of probability theory and copositive matrices*

**Minisymposium** - Methods for Modelling Correlated and Complex Data

10:20 – 10:40 T. Nummi: *Analysis of multivariate growth curves with smoothing splines*
10:40 – 11:00 L. Shi: *Robust estimation in meta-regression analysis*
11:00 – 11:20 I. Ngaruye: *Small Area Estimation for multivariate repeated measures data*

**11:20 – 11:40 Coffee Break**

**Minisymposium** - Inequalities in Matrix Theory and Probability

11:40 – 12:00 K. Castillo: *On some inequalities for eigenvalues of a special class of unitary matrices*
12:00 – 12:20 M. Mattila: *Defining positive definite arithmetical functions and a partial order on the set of arithmetical functions by using matrix inequalities*
12:20 – 12:40 F. R. Rafaeli: *Inequalities of zeros of classical orthogonal polynomials via Jacobi matrices*

**12:40 – 14:10 Lunch**

**Plenary Session**

14:10 – 14:50 R. Kala: *A new look at combining information from stratum submodels*

**Minisymposium** - Estimation and Testing in Linear Models

15:00 – 15:20 R. Zmyślony: *Application of Jordan algebra for statistical inference in multivariate normal models*
15:20 – 15:40 K. Filipiak: *Estimation of parameters under a generalized growth curve model*
15:40 – 16:00 F. Carvalho: *Best quadratic unbiased estimators for variance components in models with orthogonal block structure*

**16:00 – 16:20 Coffee break**

**Contributed Session III**

16:20 – 16:40 C. Santos: *On the extension of a balanced mixed model*
16:40 – 17:00 A. C. Carapito: *Lyapunov-Metzler inequalities with solutions sharing a common Schur complement*

17:00 - **Poster Session**

V. Kopčová: *Unbiased estimator using hypergeometric function*
I. Sousa-Ferreira: *Hybrid model for recurrent event data*

## Wednesday, June 8, 2016

### Plenary Session

9:30 – 10:10 A. Agresti: *Some perspectives about generalized linear modeling*

### Contributed Session IV

10:20 – 10:40 S. Haslett: *Parameter inestimability in hierarchical log-linear models for sparse contingency tables*

### Special Session - part A

Session devoted to the $75^{\text{th}}$ birthday of Professor Jeffrey J. Hunter

10:40 – 11:20 J. J. Hunter: *A fifty year journey with colleagues, generalized matrix inverses and applied probability*

**11:20 – 11:40 Coffee break**

### Contributed Session V

11:40 – 12:00 E. Fišerová: *Conics and quadric surfaces fitting to correlated data*

12:00 – 12:20 C. Dias: *Inference with* **vec** *type operators*

12:20 – 12:40 C. Nunes: *Comparing for one-way fixed effects models the usual and the random sample sizes ANOVA*

**12:40 – 14:00 Lunch**

### Special Session - part B

Session devoted to the $75^{\text{th}}$ birthday of Professor Jeffrey J. Hunter

14:00 – 14:30 T. Dayar: *Representing probability vectors compactly*

14:30 – 15:00 S. Kirkland: *Kemeny's constant and an analogue of Braess' paradox for Markov chains*

15:00 – 15:30 G. Latouche: *The deviation matrix and quasi-birth-and-death processes*

15:30 – 16:00 P. G. Taylor: *The Markov-modulated Erlang Loss System*

### 16:00 - EXCURSION

### 19:00 - Conference Dinner

## Thursday, June 9, 2016

### Plenary Session

9:30 – 10:10 R. A. Bailey: *Association schemes in designed experiments*

### Contributed Session VI

10:20 – 10:40 D. Kokol Bukovšek: *Seeking for a joint pmf given the sum of the marginal distributions*
10:40 – 11:00 C. Andrade: *Statistical analysis in climate research: aridity conditions in the Iberian Peninsula - a case study*
11:00 – 11:20 M. F. Teodoro: *Modeling the caregivers knowledge about pediatric hypertensions*

**11:20 – 11:40 Coffee break**

**11:40 – Special Session**

The Memorial Session of Ingram Olkin

**13:00 – 14:10 Lunch**

### Contributed Session VII

14:10 – 14:30 A. Correia: *Confirmatory factor analysis for Entrepreneurial Framework Conditions*
14:30 – 14:50 R. Covas: *Variance-covariance matrix estimation in double multivariate data with symmetric monotone missing values*

14:50 – *Closing*

# Part III

# Minisymposia

# Magic Squares, Prime Numbers and Postage Stamps

**Ka Lok Chu[1] and George P. H. Styan[2]**

[1] Dawson College, Westmount (Québec), Canada
[2] McGill University, Montréal (Québec), Canada

Talks in this minisymposium:

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- Se-tenant philatelic Machin-definitive blocks with selected total face-values, with special emphasis on the Royal Mail "Stamps for Cooks" Prestige Booklet
  *by*
  Nathan Hin Shun Chu (Canada)
  Ka Lok Chu (Canada)
  George P. H. Styan (Canada)

- A sensational $7 \times 7$ pandiagonal magic square with non-consecutive entries and "diamond-square arrangement" matrices for knight's tours in a pandiagonal magic carpet
  *by*
  George P. H. Styan (Canada)
  Walter Trump (Germany)
  Ka Lok Chu (Canada)
- Some comments on Margaret Kepner's "Magic Square 25 Study" (2010)
  *by*
  Reijo Sund (Finland)
  Ka Lok Chu (Canada)
  George P. H. Styan (Canada)
- Some comments on Sophie Germain prime numbers and on two philatelic magic-carpet dinner-placemats for the IWMS-2016 Madeira Magic Minisymposium
  *by*
  Ka Lok Chu (Canada)
  George P. H. Styan (Canada)

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# Multivariate Linear Models

**Katarzyna Filipiak**

Poznań University of Technology, Poland

## Abstract

Multi-level multivariate data, where the observations are collected on more than one variable, at different locations, repeatedly over time, and at different depths etc. are booming in all disciplines in the 21st century. One of the main problem is to model and analyze such a multivariate data. Therefore the goal of this session is to present recent results on estimation and testing of unknown parameters under various multi-level multivariate models, especially models with a structured mean or variance-covariance matrix.
The results on determination of some estimators of unknown parameters, on characterization of their properties and on comparison of proposed estimators, as well as procedures of testing hypotheses devoted to structured mean or variance-covariance matrix are mostly welcome to this session.

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**Invited speakers:**

- Miguel Fonseca (Portugal)
  *Estimation for the growth curve model with orthogonal covariance structure*
- Daniel Klein (Slovakia)
  *Testing mean under compound symmetry covariance setup*
- Ivan Žežula (Slovakia)
  *Comparison of estimators in a multivariate linear model with generalized uniform correlation structure*

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# Inequalities in Matrix Theory and Probability

**Alexander Kovačec**

Universidade de Coimbra, Portugal

## Abstract

The themes of this session are bound together by the words 'matrix' and 'inequalities'.

$*$ Square matrices whose $i, j$-entries are defined by number-theoretical constructs have fascinated many researchers at least since H.J.S. Smith proved in 1876 that if $S = \{x_1, x_2, ..., x_n\}$ is a factor closed set of integers, and $f : \mathbb{Z}^+ \to \mathbb{R}$ any arithmetical function, then the determinant of the matrix $[f(\gcd(x_i, x_j))]_{i,j=1}^n$ can be expressed via the Möbius function $\mu$ and Dirichlet convolution $*$ by the surprisingly simple formula $\prod_{k=1}^n (f * \mu)(x_k)$. Professor Mika Mattila from Tampere will speak on what it means for such a (necessarily symmetric) matrix that it is positive semidefinite; i.e. satisfies $x^*[f(\gcd(x_i, x_j))]x \geq 0$ for all $x \in \mathbb{C}^n$.

$*$ A family $\{p_n(x)\}_{n\geq 0}$ of nonzero one-variable polynomials with $\deg p_n = n$ is called an *orthogonal family* with respect to a nonnegative, on an interval $[a, b]$ Lebesgue-integrable weight function $w(x)$, if, whenever $n \neq m$, then there holds $\int_a^b p_n(x)p_m(x)w(x)dx = 0$. A famous example is given by the Jacobi-polynomials $P_n^{\alpha,\beta}(x)$ defined w.r.t. the interval $[-1, 1]$ and the weight function $w(x) = (1-x)^\alpha(1+x)^\beta$. Special cases go by the name of Tschebychef, Gegenbauer, and Legendre. Other families are due to Laguerre and Hermite. The zeros of such classical orthogonal polynomials have interesting interlacing properties and it seems from their abstracts that Drs Rafaeli and Castillo have found a new approach to provide information about the monotonicity of these zero sets (as evolving with $n$) by using a theorem that also plays a role in quantum mechanics and which gives information about the eigenvalues of certain - often infinite - tridiagonal matrices, called Jacobi matrices.

$*$ The fruitful study of orthogonality on intervals could not fail to induce the idea to study extensions and generalizations of classical orthogonality to other domains. One such extension concerns orthogonality on the unit circle. We will be informed on the history of the subject by Dr. Castillo who currently is Pos-Doc at the University of Coimbra where, together with Professor Petronilho, he gave new results on the interlacing properties of such polynomials by finding that the correct analogue to Jacobi matrices for the interval case are, for the circle case, certain unitary matrices.

$*$ The last talk has a curious origin. The speaker is a collaborator in the Delfos project which aims to develop the talents of mathematically interested high school students. In Delfos' online facility for problem solving, the Forum, an ex-student of Delfos once challenged the readers - without further comments - to show a certain inequality of the form $\text{Prob}(|X-Y| \leq b) \leq c\text{Prob}(|X-Y| \leq a)$, where $X, Y$ are independent, identically distributed vector valued random variables. Having at about that time solved a probabilistic problem for a friend of his, the speaker got interested in the challenge and solved it for the finite case by working with real symmetric $n \times n$ matrices $C$ satisfying $x^T Cx \geq 0$ whenever $x \in \mathbb{R}^n_{>0}$. Such matrices are called *copositive*. Upon publicizing his solution in the Forum the speaker was informed that it was considered a 'difficult exercise' in a book by Noga Alon on probabilistic methods in combinatorics, and actually has as base a paper by Alon and Yuster. After verifying the suspicion that Alon and Yuster proved their result in quite a different manner, the speaker invited Delfos students to work with him on proving the remaining Alon-Yuster results with the matrix method proposed. Two students, Miguel Moreira, and David P. Martins took up the challenge and the outcome of this collaboration is what will be presented.

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**Invited speakers:**

- Kenier Castillo (Portugal)
  *On some inequalities for eigenvalues of a special class of unitary matrices*
- Mika Mattila (Finland)
  *Defining positive definite arithmetical functions and a partial order on the set of arithmetical functions by using matrix inequalities*
- Fernando Rodrigo Rafaeli (Brasil)
  *Inequalities of zeros of classical orthogonal polynomials via Jacobi matrices*

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# Methods for Modelling Correlated and Complex Data

**Jianxin Pan**

University of Manchester, UK

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**Invited speakers:**

- Innocent Ngaruye (Sweden)
  *Small Area Estimation for multivariate repeated measures data*
- Tapio Nummi (Finland)
  *Analysis of multivariate growth curves with smoothing splines*
- Lei Shi (China)
  *Robust estimation in meta-regression analysis*

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# Estimation and Testing in Linear Models

**Roman Zmyślony**

University of Zielona Góra, Poland

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**Invited speakers:**

- Francisco Carvalho (Portugal)
  *Best quadratic unbiased estimators for variance components in models with orthogonal block structure*
- Ricardo Covas (Portugal)
  *Variance-covariance matrix estimation in double multivariate data with symmetric monotone missing values*
- João T. Mexia (Portugal)
  *Normal approximations to noncentral Wishart matrices*

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Part IV

# Invited Speakers

# Some perspectives about generalized linear modeling

**Alan Agresti**

University of Florida, Gainesville, Florida, USA

## Abstract

This talk discusses several topics pertaining to generalized linear modeling. With focus on categorical data, the topics include (1) bias in using ordinary linear models with ordinal categorical response data, (2) interpreting effects with nonlinear link functions, (3) cautions in using Wald inference (tests and confidence intervals) when effects are large or near the boundary of the parameter space, and (4) the behavior and choice of residuals for GLMs. I will present few new research results, but these topics got my attention while I was writing the book 'Foundations of Linear and Generalized Linear Models', recently published by Wiley.

# Association schemes in designed experiments

**R. A. Bailey**

University of St Andrews, UK

## Abstract

Association schemes arise in designed experiments in many ways. They were first used in incomplete-block designs, but they are implicit in the treatment structure of factorial designs and in many common block structures, such as row-column designs or nested blocks. What is nice about them is the link between the matrices which show the patterns and the matrices which project onto the common eigenspaces.

In recent work, Agnieszka Lacka and I have considered designs where the treatments consist of all combinations of levels of two treatment factors and one additional control treatment. We construct nested row-column designs which have what we call control orthogonality and supplemented partial balance.

# A new look at combining information from stratum submodels

**Radosław Kala**

Poznań University of Life Sciences, Poland

## Abstract

The main principles improving the objectivity of inference from planned experiments consist on blocking the experimental units, randomizing processes, and replications of treatments on which the interest of the experimenter is focused. These principles determine a model of observations resulting from the experiment. The model with fixed effects of treatments and with random effects of various levels of blocking is classified as a mixed model. This paper deals with the issue of combining information on treatment comparisons following from several submodels induced by the randomizations involved. The approach proposed here is quite general and mainly geometrical, which simplifies the considerations.

## Keywords

Orthogonal block structure, Variance components estimation.

## References

[1] Bailey, R. A. (1981). A unified approach to design of experiments. *J. Roy. Statist. Soc. Ser. A 144*, 214–223.

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[3] Caliński, T. and Kageyama, S. (2000). *Block Designs: A Randomization Approach, Vol. I: Analysis. Lecture Notes in Statistics* **150**, Springer, New York.

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[5] Nelder, J. A. (1968). The combination of information in generally balanced designs. *J. Roy. Statist. Soc. Ser. B 30*, 303–311.

[6] Patterson, H. D. and Thompson, R. (1971). Recovery of inter-block information when the block sizes are unequal. *Biomertika 58*, 545–554.

# The 123 theorem of probability theory and copositive matrices

**Alexander Kovačec[1], Miguel Moreira[2], and David P. Martins[3]**

[1] Universidade de Coimbra, Portugal
[2] Instituto Superior Técnico, Lisbon, Portugal
[3] Oxford University, England

## Abstract

Alon and Yuster give for independent identically distributed real or vector valued random variables $X, Y$ combinatorially proved estimates of the form $\mathrm{Prob}(\|X-Y\| \le b) \le c\,\mathrm{Prob}(\|X-Y\| \le a)$. We derive these using copositive matrices instead. By the same method we also give estimates for the real valued case, involving $X+Y$ and $X-Y$, due to Siegmund-Schultze and von Weizsäcker [3] as generalized by Dong, Li and Li [2]. Furthermore we formulate a version of above inequalities as an integral inequality for monotone functions.

## Keywords

Probabilistic inequalities, Copositivity, Integral inequality.

## References

[1] Alon, N. and Yuster, R. (1995). The 123 Theorem and Its Extensions. *J. of Combin. Theory, Ser. A 72*, 321–331.
[2] Dong, Z., Li, J., and Li, W.V. (2014). A Note on Distribution-Free Symmetrization Inequalities. *J. Theor. Probab.* (DOI 10.1007/s10959-014-0538-z).
[3] Siegmund-Schultze, R. and von Weizsäcker, H. (2007). Level crossing probabilities I: One-dimensional random walks and symmetrization, *Adv. Math. 208*, 672–679.

# Joint mean-covariance modelling and its R package: jmcm

**Jianxin Pan and Yi Pan**

University of Manchester, UK

## Abstract

Longitudinal studies are commonly arising in various fields such as psychology, social science, economics and medical research, etc. It is of great importance to understand the dynamics in the mean function, covariance and/or correlation matrices of repeated measurements. However, the high-dimensionality (HD) and positive-definiteness (PD) constraints are two major stumbling blocks in modelling of covariance and correlation matrices. It is evident that Cholesky-type decomposition based methods are effective in dealing with HD and PD problems, but those methods were not implemented in statistical software yet, causing a difficulty for practitioners to use. In this talk, three Cholesky decomposition based methods for joint modelling of mean and covariance structures, namely Modified Cholesky decomposition (MCD), Alternative Cholesky decomposition (ACD) and Hyperpherical parameterization of Cholesky factor (HPC), will be introduced first. The newly developed R package jmcm which includes the MCD, ACD and HPC methods will then be introduced. Demonstration will be made by running the package jmcm and comparison of those methods will be made through analyzing two real data sets.

## Keywords

Cholesky decomposition based methods, Covariance matrix, Covariance models, R package.

# Part V

# Abstracts

# Goodness-of-fit tests for semiparametric transformation models

**James S. Allison**[1], **Marie Hušková**[2], **and Simos G. Meintanis**[3]

[1] North-West University, Potchefstroom, South Africa
[2] Charles University of Prague, Czech Republic
[3] National and Kapodistrian University of Athens, Greece

## Abstract

We consider a semiparametric model whereby the response variable following a transformation can be expressed by means of a nonparametric regression model. In this model the form of the transformation is specified analytically but incorporates an unknown transformation parameter. We develop testing procedures for the null hypothesis that this semiparametric model adequately describes the data at hand. In doing so, the test statistic is formulated on the basis of Fourier-type conditional expectations. The asymptotic distribution of the test statistic is obtained under the null as well as under alternative hypotheses. Since the limit null distribution is nonstandard, a bootstrap version is utilized in order to actually carry-out the test procedure. Monte Carlo results are included that illustrate the finite-sample properties of the new method.

## Keywords

Transformation model, Goodness-of-fit test, Nonparametric regression, Bootstrap test.

# Statistical analysis in climate research: aridity conditions in the Iberian Peninsula - a case study

**Cristina Andrade**[1,2,3] **and João Corte-Real**[3,4,5]

[1] Instituto Politécnico de Tomar, Portugal
[2] Universidade de Trás-os-Montes e Alto Douro, Vila Real, Portugal
[3] Universidade de Aveiro, Portugal
[4] Universidade Lusófona de Humanidades e Tecnologias, Lisboa, Portugal
[5] Universidade de Évora, Portugal

## Abstract

Researchers in atmospheric sciences often use the popular format named Network Common Data Form (NetCDF) developed by University Corporation for Atmospheric research (UCAR) to create, manage, store and distribute scientific data. It is a platform independent format, available for several operational systems, and it was designed to represent multidimensional, array-oriented scientific data. Usually an array has two dimensions (2D), in atmospheric sciences that can means a temperature, precipitation or pressure field given certain coordinates: latitude and longitude. Arrays having more than two dimensions, e.g., when to the previous fields it is added altitude (3D) or even time (4D) these arrays are called multidimensional arrays. Programming and work with multidimensional data can be challenging, although NetCDF data is self-describing and support direct access to small subset or larger datasets (since storage is made as arrays). Consequently some common statistical analysis can still be performed in climate research but from another view point [5], [6], [7].

Aridity plays a key role to characterize the climate of a region, since it has a major impact on water resources and human activities. In this case study, several statistical methods are going to be applied to an aridity index, the De Martonne aridity index [4] between 1901 and 2012 in the Iberian Peninsula. Gridded precipitation totals and air temperature datasets are used on a monthly basis to compute this index. Results revealed that climate was subjected to both spatial and temporal variabilities and statistically significant trends were detected [1], [2]. A regional division of the Iberian Peninsula according to aridity conditions was attained by a hierarchical cluster analysis and is going to be presented. The selection of the clusters following Ward method [5] showed high spatial coherence, and allowed the study of the general spatial behavior of aridity conditions in Iberia during this period. These results are in clear accordance with some outcomes achieved by [2], [3] regarding other climatic indices.

## Keywords

Multivariate Statistics, Climatic indices, De Martonne Aridity Index, Iberian Peninsula.

## Acknowledgements

We acknowledge the National Center for Atmospheric Research Staff for the Climate Data Guide: CRU TS3.21 Gridded precipitation and other meteorological variables since 1901, retrieved from
https://climatedataguide.ucar.edu/climate-data/cru-ts321-gridded-precipitation-and-other-meteorological-variables-1901.

This work is supported by: European Investment Funds by FEDER/COMPETE/POCI – Operational Competitiveness and Internationalization Program, under Project $POCI-01-0145-FEDER-006958$ and National Funds by FCT–Portuguese Foundation for Science and Technology, under the project $UID/AGR/04033/2013$.

## References

[1] Andrade, C. and Corte-Real, J. (2016). Aridity conditions in the Iberian Peninsula during the XX century. *Int. J. of Environ. Sci. Vol. I*, 52–58, ISSN: 2367-8941.

[2] Andrade, C. and Corte-Real, J. (2016). Assessment of the spatial distribution of continental-oceanic climate indices in the Iberian Peninsula. *Int. J. Climatol.*, doi: 10.1002/joc.4685 (published online).

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# Lyapunov-Metzler inequalities with solutions sharing a common Schur complement

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## Abstract

Given a set of Lyapunov inequalities $A_i^T P_i + P_i A_i < 0$, with $i = 1, \cdots, N$, such that $A_1, A_2, \cdots, A_N$ are Metzler square matrices, we investigate when the Lyapunov solutions $P_i$, with $i = 1, \cdots, N$ share the same Schur complement of certain order. In view of the results obtained in [1], this provides a sufficient condition for stabilizability by partial reset of positive switched linear systems under arbitrary switching law.

## Keywords

Metzler matrix, Schur complement, Stability, Switched system, Quadratic Lyapunov function.

## Acknowledgements

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# Best quadratic unbiased estimators for variance components in models with orthogonal block structure

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## Abstract

Quasi-normality is usually assumed in deriving estimators for variance components. This entails "fixing" the weight of the queues since we then assume $\mu_4 = 3(\sigma^2)^2$. This is a rather strong assumption when, as usual, we are obtaining quadratic estimators. We will overcome this restriction imposing lighter conditions on the fourth order moments and obtaining the corresponding best quadratic unbiased estimators for models with orthogonal block structures.

## Keywords

Mixed models, Orthogonal block structure models, Completeness.

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# On some inequalities for eigenvalues of a special class of unitary matrices

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## Abstract

The purpose of this talk is twofold. First, to present new information on the historical development of some results on zeros of paraorthogonal polynomials on the unit circle. Second, to obtain some known and new interlacing properties of their zeros– as an eigenvalue problem for certain unitary matrices which are the "right" unitary analogue of Jacobi matrices– by using, exclusively, a result form matrix theory due to Arbenz and Golub [1].

## Keywords

Paraorthogonal polynomials on the unit circle, Unitary matrices, Eigenvalues, Rank one perturbations.

## References

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# Se-tenant philatelic Machin-definitive blocks with selected total face-values, with special emphasis on the Royal Mail "Stamps for Cooks" Prestige Booklet

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## Abstract

Even though Royal Mail has issued several hundred Machin-definitive stamps, it seems that they have not issued a single Machin-definitive stamp with face value $21p$. From the booklet panes in the 1969 Royal Mail "Stamps for Cooks" £1 Prestige Booklet


*Scott* #BK125/126, image #BC21 [*sic*], we identify 255 "se-tenant" philatelic Machin-definitive blocks with combined face value $21d$. Se-tenant stamps are printed from the same plate and sheet and adjoin one another, unsevered in a strip or block. We have also identified several se-tenant philatelic Machin-definitive blocks for various face-values greater than $21p$ and for which it seems that Royal Mail has not issued a single Machin-definitive stamp. We use the symbol $p$ to denote decimal-pence (introduced in 1971) and $d$ (from *denarius*) pre-decimal pence with $240d = £1$.

# Some comments on Margaret Kepner's "Magic Square 25 Study" (2010)

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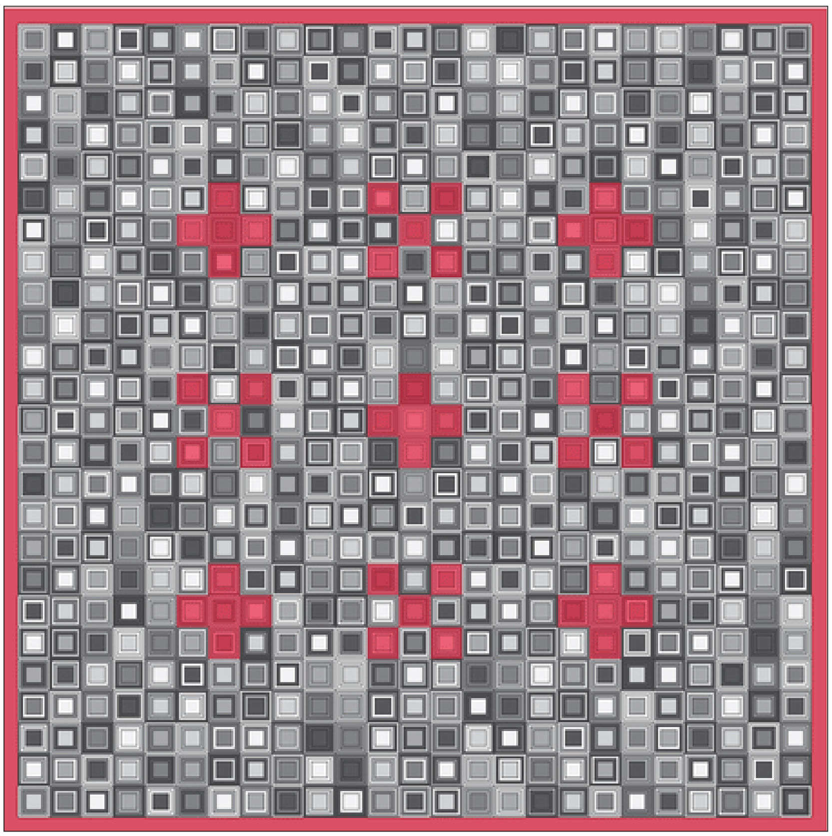
## Abstract



Margaret Kepner received the First Prize Award for her "Magic Square 25 Study" (2010) archival inkjet print (copy displayed above) at the 2011 Joint Mathematics Meetings in New Orleans [*Journal of Mathematics and the Arts,* vol. 5, no. 3, p. 148 (Figure 2), September 2011].

As observed in

"Mathematical Imagery":
`http://www.ams.org/mathimagery/displayimage.php?pid=346`

> The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Maurits Cornelis Escher (1898–1972), usually referred to as M. C. Escher, was a Dutch graphic artist. He is known for his often mathematically inspired woodcuts, lithographs, and mezzotints. These feature impossible constructions, explorations of infinity, architecture, and tessellations. represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works. Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics — origami, computer-generated landscapes, tesselations, fractals, anamorphic art, and more.

We believe that Kepner's "Magic Square 25 Study" corresponds to the $25 \times 25$ "Kepner matrix", $\mathbf{K}$, defined as:

$$
\begin{pmatrix}
405 & 118 & 301 & 514 & 222 & 62 & 270 & 583 & 166 & 479 & 344 & 527 & 235 & 448 & 6 & 621 & 184 & 392 & 75 & 288 & 128 & 461 & 49 & 357 & 565 \\
561 & 149 & 457 & 40 & 353 & 218 & 401 & 114 & 322 & 505 & 495 & 58 & 266 & 579 & 162 & 2 & 335 & 548 & 231 & 444 & 284 & 617 & 175 & 388 & 96 \\
92 & 275 & 613 & 196 & 384 & 374 & 557 & 140 & 453 & 36 & 501 & 214 & 422 & 105 & 318 & 158 & 491 & 54 & 262 & 595 & 435 & 23 & 331 & 544 & 227 \\
248 & 431 & 19 & 327 & 535 & 375 & 88 & 296 & 609 & 192 & 32 & 365 & 553 & 136 & 474 & 314 & 522 & 205 & 418 & 101 & 591 & 154 & 487 & 70 & 258 \\
254 & 587 & 170 & 483 & 66 & 531 & 244 & 427 & 10 & 348 & 188 & 396 & 84 & 292 & 600 & 465 & 28 & 361 & 574 & 132 & 122 & 305 & 518 & 201 & 414 \\
599 & 157 & 490 & 53 & 261 & 226 & 439 & 22 & 330 & 543 & 383 & 91 & 279 & 612 & 195 & 35 & 373 & 556 & 144 & 452 & 317 & 500 & 213 & 421 & 109 \\
100 & 313 & 521 & 209 & 417 & 257 & 590 & 153 & 486 & 74 & 539 & 247 & 430 & 18 & 326 & 191 & 379 & 87 & 295 & 608 & 473 & 31 & 369 & 552 & 135 \\
131 & 469 & 27 & 360 & 573 & 413 & 121 & 309 & 517 & 200 & 65 & 253 & 586 & 174 & 482 & 347 & 530 & 243 & 426 & 14 & 604 & 187 & 395 & 83 & 291 \\
287 & 620 & 183 & 391 & 79 & 569 & 127 & 460 & 48 & 356 & 221 & 409 & 117 & 300 & 513 & 478 & 61 & 274 & 582 & 165 & 5 & 343 & 526 & 239 & 447 \\
443 & 1 & 339 & 547 & 230 & 95 & 283 & 616 & 179 & 387 & 352 & 560 & 148 & 456 & 44 & 509 & 217 & 400 & 113 & 321 & 161 & 499 & 57 & 265 & 578 \\
13 & 346 & 534 & 242 & 425 & 290 & 603 & 186 & 399 & 82 & 572 & 130 & 468 & 26 & 364 & 204 & 412 & 120 & 308 & 516 & 481 & 69 & 252 & 585 & 173 \\
169 & 477 & 60 & 273 & 581 & 446 & 9 & 342 & 525 & 238 & 78 & 286 & 624 & 182 & 390 & 355 & 568 & 126 & 464 & 47 & 512 & 220 & 408 & 116 & 304 \\
320 & 508 & 216 & 404 & 112 & 577 & 160 & 498 & 56 & 269 & 234 & 442 & 0 & 338 & 546 & 386 & 99 & 282 & 615 & 178 & 43 & 351 & 564 & 147 & 455 \\
451 & 39 & 372 & 555 & 143 & 108 & 316 & 504 & 212 & 420 & 260 & 598 & 156 & 494 & 52 & 542 & 225 & 438 & 21 & 334 & 199 & 382 & 90 & 278 & 611 \\
607 & 190 & 378 & 86 & 299 & 139 & 472 & 30 & 368 & 551 & 416 & 104 & 312 & 520 & 208 & 73 & 256 & 594 & 152 & 485 & 325 & 538 & 246 & 434 & 17 \\
177 & 385 & 98 & 281 & 619 & 459 & 42 & 350 & 563 & 146 & 111 & 324 & 507 & 215 & 403 & 268 & 576 & 164 & 497 & 55 & 545 & 233 & 441 & 4 & 337 \\
333 & 541 & 229 & 437 & 20 & 610 & 198 & 381 & 94 & 277 & 142 & 450 & 38 & 371 & 559 & 424 & 107 & 315 & 503 & 211 & 51 & 264 & 597 & 155 & 493 \\
489 & 72 & 255 & 593 & 151 & 16 & 329 & 537 & 245 & 433 & 298 & 606 & 194 & 377 & 85 & 550 & 138 & 471 & 34 & 367 & 207 & 415 & 103 & 311 & 524 \\
515 & 203 & 411 & 124 & 307 & 172 & 480 & 68 & 251 & 589 & 429 & 12 & 345 & 533 & 241 & 81 & 294 & 602 & 185 & 398 & 363 & 571 & 134 & 467 & 25 \\
46 & 359 & 567 & 125 & 463 & 303 & 511 & 224 & 407 & 115 & 580 & 168 & 476 & 64 & 272 & 237 & 445 & 8 & 341 & 529 & 394 & 77 & 285 & 623 & 181 \\
366 & 554 & 137 & 470 & 33 & 523 & 206 & 419 & 102 & 310 & 150 & 488 & 71 & 259 & 592 & 432 & 15 & 328 & 536 & 249 & 89 & 297 & 605 & 193 & 376 \\
397 & 80 & 293 & 601 & 189 & 29 & 362 & 570 & 133 & 466 & 306 & 519 & 202 & 410 & 123 & 588 & 171 & 484 & 67 & 250 & 240 & 428 & 11 & 349 & 532 \\
528 & 236 & 449 & 7 & 340 & 180 & 393 & 76 & 289 & 622 & 462 & 45 & 358 & 566 & 129 & 119 & 302 & 510 & 223 & 406 & 271 & 584 & 167 & 475 & 63 \\
59 & 267 & 575 & 163 & 496 & 336 & 549 & 232 & 440 & 3 & 618 & 176 & 389 & 97 & 280 & 145 & 458 & 41 & 354 & 562 & 402 & 110 & 323 & 506 & 219 \\
210 & 423 & 106 & 319 & 502 & 492 & 50 & 263 & 596 & 159 & 24 & 332 & 540 & 228 & 436 & 276 & 614 & 197 & 380 & 93 & 558 & 141 & 454 & 37 & 370
\end{pmatrix}
$$

which we find has several interesting properties [Report 2013-05 from the Department of Mathematics and Statistics, McGill University]. In particular, the Kepner matrix $\mathbf{K}$ has rank 17 and nullity 8, and is composite, pandiagonal, bimagic, and EP.

# Confirmatory factor analysis for Entrepreneurial Framework Conditions

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## Abstract

Entrepreneurship is increasingly recognised as an essential component of economic growth, employment generation, innovation as well as socio-economic development (OECD 2003). Global Entrepreneurship Monitor (GEM) is a large scale database for internationally comparative entrepreneurship that includes information about many aspects of entrepreneurship activities of a large number of countries. This project has two main sources of primary data: the Adult Population Survey (APS) and the National Expert Survey (NES). NES provides detailed information about entrepreneurship activities and its model suggests that the different institutional environments (economic, political and social) create different Entrepreneurial Framework Conditions (EFCs) that may vary among different types of economies and may change along with economic development. The GEM model defines 12 basic EFCs modelling entrepreneurship dynamics in economies: Financial environment; Governmental policies; Governmental programs; Entrepreneurial education and training; R&D transfer; Commercial and professional infrastructure; Internal market openness; Physical and services infrastructure; and Social and cultural norms (GEM, 2011).

In this work the 2011 National Expert Survey dataset, second to last available on the project website, is studied. Our goal is to test the structure proposed by GEM for EFC's, using Confirmatory Factor Analysis (CFA). Unlike Exploratory factor analysis (EFA), CFA produces many goodness-of-fit measures to evaluate the model but do not calculate factor scores. CFA is a special case of the structural equation model (SEM), also known as the covariance structure (McDonald, 1978) or the linear structural relationship (LISREL) model (Jöreskog & Sörbom, 2004). Goodness-of-fit statistics obtained with the original structure $\chi^2$ is 5400.242 which is so large that the null hypothesis of a good fit is rejected at the .05 level ($p<.000$). The degrees of freedom is 1208. Root Mean Square Error of Approximation (RMSEA) 0.043 is not large enough to reject the null hypothesis ($p=1{,}000$). Comparative Fit Index (CFI) 0.895 is small. Therefore, this factor model shows a poor fit and needs to be modified somehow. The modifications needed for this dataset are presented and then is tested in 2012 National Expert Survey dataset, last available on the project website.

## Keywords

Multivariate statistical analysis, Environmental data, Water quality, Reservoirs, MANOVA.

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# Variance-covariance matrix estimation in double multivariate data with symmetric monotone missing values

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## Abstract

In [1] a list of missing data patterns for variance-covariance matrices is given. These are believed to be the ones of most practical interest and they have been tackled in the literature by different approaches. For us, the one of most interest is the monotone missing value problem, also known as the *staircase* missing data. There is some literature on the subject, of which [2] and [3] we cite as examples.
We introduce a new case of missing data, a bit more general then the monotone missing value problem but of immense interest in financial markets. In this case the covariance-variance matrix has symmetric monotone missing values, *i.e.*, missing values in both triangular parts of the variance-covariance matrix.

## Keywords

Variance-covariance matrix, Multivariate statistics, Missing data.

## References

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# Representing probability vectors compactly*

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## Abstract

The transition rate matrix associated with a multi-dimensional Markov chain having a relatively large reachable state space [3] can be represented compactly using Kronecker products [1]. Nevertheless, probability vectors employed in the numerical analysis of such representations are still proportional to the size of the reachable state space. As the number of dimensions increases, this size increases exponentially, and therefore, poses a challenge. The current talk shows that it is possible to store probability vectors during numerical analysis relatively compactly using higher-order singular value decomposition [4]. Yet, the basic operation of vector-Kronecker product multiplication [2] can still be performed relatively efficiently. Furthermore, larger space savings are obtained as the number of dimensions increases.

## Keywords

Markov chains, Reachable state space, Kronecker products, Higher-order singular value decomposition.

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# Inference with vec type operators

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## Abstract

In this work we consider models of the form $\mathbf{M} = \boldsymbol{\mu} + \overline{\mathbf{E}}$. These models have degree k and can be applied to symmetric stochastic matrices. The development of the models is based on spectral analysis of the respective average matrices. We also show how to use the operators of the type **vec** in the validation of the model. These operators enable us to present results that allow to perform inference for isolated matrices and structured families of matrices.

## Keywords

Models for symmetric stochastic matrices, vec type operators, Structured families.

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# Adding observations in regression analysis

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## Abstract

We consider the following situation: In a regression model the least squares estimator of the regression parameter is computed. Some new observations are added to the original observations. What is an efficient method to update the regression parameter estimators?
One method is the matrix inversion-method due to Törnquist. This, however, will only work if very few observations are added. A more efficient method consists in forming the Gram-Schmidt orthogonalizers and computing a linearly sufficient statistic from them.
An additional scaling procedure will finally read to a new regression model. In this model least squares estimation can either again be done by a computer or by developing new estimation formulae.

# Estimation of parameters under a generalized growth curve model

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[2] P. J. Šafárik University, Košice, Slovakia

## Abstract

Let us consider an experiment, in which $p$ characteristics are observed in $q$ time points for each of $n$ treatments. The data from such an experiment are arranged in three-indices matrix (tensor of order three) and can be modeled using a generalize growth curve model

$$\mathcal{Y} = (\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X} + \mathcal{E},$$

where $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X}$ is a product of tensor $\mathcal{X}$ from each of three "sides" by matrices $\boldsymbol{A} \in \mathbb{R}^{n \times n_1}$, $\boldsymbol{B} \in \mathbb{R}^{p \times p_1}$ i $\boldsymbol{C} \in \mathbb{R}^{q \times q_1}$ respectively, i.e.,

$$((\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})\mathcal{X})_{kij} = \sum_{\alpha=1}^{n_1} \sum_{\beta=1}^{p_1} \sum_{\gamma=1}^{q_1} a_{k\alpha} b_{i\beta} c_{j\gamma} x_{\alpha\beta\gamma};$$

cf. Savas and Lim (2008).
Assuming independence of treatments, it is natural to study a doubly-separable variance-covariance matrix of the tensor of observations, which can be presented as a Kronecker product of three matrices, where one of these matrices is identity of order $n$. The aim of this paper is to determine the maximum likelihood estimators of unknown parameters (expectation and variance-covariance matrix) under a generalized growth curve model.
Presented results are some generalization of the paper by Srivastava et al. (2009).

## Keywords

Generalized growth curve model, Maximum likelihood estimates, Block-trace operator, Partial-trace operator.

## References

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# Conics and quadric surfaces fitting to correlated data

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## Abstract

Fitting quadratic curves and quadric surfaces to given data points is a fundamental task in many fields like engineering, astronomy, physics, biology, quality control, image processing, etc. The classical approach for fitting is geometric fit based on minimization of geometric distances from observed data points to the fitted curve/surface. In the contribution, we focus on solving the problem of geometric fit to correlated data using the linear regression model with nonlinear constraints. The constraints are represented by the general equation of the certain curve/surface. In order to obtain approximate linear regression model, these nonlinear constraints are being linearized by the first-order Taylor expansion. The iterative estimation procedure provides locally best linear unbiased estimates of the unknown algebraic parameters of the considered curve/surface together with unbiased estimates of variance components. Consequently, estimates of geometric parameters, volume, surface area, etc. and their uncertainties can be determined.

## Keywords

Geometric fitting, Least squares, Variance components, Accuracy, Conics, Quadric surfaces.

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# Estimation for the growth curve model with orthogonal covariance structure

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## Abstract

The growth curve model is a well documented multivariate model in literature, with a well established methodology of maximum likelihood estimation. We propose a growth curve model family with an orthogonal covariance structure for lines and columns, proceeding with the derivation of maximum likelihood statistics. Many familiar models fall within this model family, as it will be shown.

## Keywords

Growth curve model, Orthogonal covariance structure, Maximum likelihood.

## Acknowledgements
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# Hadamard matrices on error detection and correction: Useful links to BIBD

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## Abstract

In the areas of Computer Science and Telecommunications there is a huge amount of applications in which error control, error detection and error correction are crucial tools to enable reliable delivery of digital data over unreliable communication channels, thus providing quality of service. Hadamard matrices can almost directly be used as an error-correcting code using a Hadamard code, generalized in Reed-Muller codes. Advances in algebraic design theory by using deep connections with algebra, finite geometry, number theory, combinatorics and optimization provided a substantial progress on exploring Hadamard matrices. Their construction and its use on combinatorics are crucial nowadays in diverse fields such as: quantum information, communications, networking, cryptography, biometry and security. Hadamard Matrices give rise to a class of block designs named Hadamard configurations and different applications of it based on new technologies and codes of figures such as QR Codes are present almost everywhere. Some connections to Balanced Incomplete Block Designs are very well known as a tool to solve emerging problems in these areas. We will explore the use of Hadamard Matrices on QR Codes error detection and correction. Some examples will be provided.

## Keywords

BIBD, Block designs, Hadamard matrices, QR Codes, Reed-Muller codes.

## Acknowledgements

The authors were partially sponsored by Fundação para a Ciência e a Tecnologia, Portugal, through the project UID/MAT/00006/2013.

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# Parameter inestimability in hierarchical loglinear models for sparse contingency tables

**Stephen Haslett**

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## Abstract

Parameter inestimability in hierarchical loglinear models for sparse complete multidimensional contingency tables where there are margins containing zeros has been discussed in [2]. There is also the possibility of parameter inestimability without any marginal zeros - the simplest case is the 2x2x2 table with zeros in the (1,1,1) and (2,2,2) cells discussed in [1]. This presentation will explore how the problem of inestimable parameters even without marginal zeros generalises to tables with more than two categories per variable, or greater than three dimensions.

## Keywords

Estimability, Internal zeros, Hierarchical models, Loglinear models, Marginal zeros, Seed zeros, Sparse contingency tables.

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# A fifty year journey with colleagues, generalized matrix inverses and applied probability

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## Abstract

The presenter has over his career made a number of contributions to a range of problems involving the application of generalized matrix inverses to the theory of Markov chains and semi-Markov processes. Along this journey many individuals, colleagues and friends, have had an impact on his career. This presentation highlights some of the influences that have impacted on his main contributions. We explore a number of key results that appear in some of his published work in these fields of research.

## Keywords

Generalized matrix inverses, Markov chains.

# Kemeny's constant and an analogue of Braess' paradox for Markov chains

**Steve Kirkland**

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## Abstract

A square matrix that is entrywise nonnegative and has all row sums equal to 1 is called a stochastic matrix, and such matrices play a central role in the study of Markov chains. Given a stochastic matrix $A$, Kemeny's constant $K(A)$ measures the expected number of steps required for the Markov chain to transition from a given initial state to a randomly chosen final state.
In this talk, we give a brief introduction to Kemeny's constant. We will then explore an analogue of Braess' paradox (wherein adding a road to a network can have the counter-intuitive effect of increasing travel times). Specifically, we will discuss how adding an edge into an undirected graph can increase the value of Kemeny's constant for a certain Markov chain that is naturally associated with the graph.

## Keywords

Kemeny's constant, Stochastic matrix, Random walk on a graph.

# Testing mean under compound symmetry covariance setup

**Ivan Žežula[1], Daniel Klein[1], Anuradha Roy[2], and Ricardo Leiva[3]**

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[2] The University of Texas at San Antonio, USA
[3] F.C.E. Universidad Nacional de Cuyo, Argentina

## Abstract

Testing the mean vector assuming compound (or block compound) symmetry covariance structure in one-level (or two-level) multivariate data is discussed. This problem was already discussed e.g. by Geisser (1963) or Szatrowski (1982). Recently, Roy et. al (2015) arrived to the solution of this problem as a natural extension of the Hotelling's $T^2$ test statistic, which seemed to be very easy; the solution was obtained via orthogonal transformation which diagonalize (or block-diagonalize) the covariance matrix of the transformed data. Natural question arose afterwards: Is the solution independent to this transformation? We will discuss this question.

## Keywords

Compound symmetry, Block compound symmetry, Block $T^2$ statistic, Hotelling's $T^2$ statistic, Lawley-Hotelling trace distribution.

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# Seeking for a joint pmf given the sum of the marginal distributions

**Damjana Kokol Bukovšek, Matjaž Omladič, and Gregor Šega**

University of Ljubljana, Slovenia

## Abstract

When a discussion of the exchange rates among currencies of several countries is given, the usual approach is to study the rates of all these currencies towards the currency of a given country, sometimes called the numeraire currency, and then compute their bilateral relations from there. It is our aim to discuss possible quantitatively based backgrounds on the exchange rates of countries without giving any of them a special role. For instance, if we take three comparable countries $A$, $B$ and $C$ and denote by $X$, resp. $Y$, resp. $Z$, the logarithm of the nominal exchange rate of currency of the country $B$, resp. $C$, resp. $A$, in terms of the currency of country $A$, resp. $B$, resp. $C$, then we clearly must have $X + Y + Z = 0$. It is the aim of this paper to apply linear and multilinear algebra theory in order to shed some light on these and similar questions.

# The deviation matrix and quasi-birth-and-death processes

**Guy Latouche**

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## Abstract

The deviation matrix is defined as $D = \sum_{n\geq 0}(P^n - \mathbf{1}\,\boldsymbol{\pi}^{\mathrm{t}})$ where $P$ is the transition matrix of an irreducible, positive recurrent Markov chain, and $\boldsymbol{\pi}$ is its stationary probability vector. It is closely related to the equation $(I-P)\boldsymbol{x} = \boldsymbol{r} + w\mathbf{1}$, known as Poisson's equation, where $\boldsymbol{r}$ is a given vector, and it plays an important role in the analysis of Markov chains: one may recall its connections to the sensitivity analysis of the stationary distribution of a Markov chain, and to the Central Limit theorem for Markov chains. If the state space is finite, then the deviation matrix is the group inverse of $I-P$ in discrete time; in continuous time, it is the group inverse of the generator. As is often the case in Markov chains theory, the deviation matrix may be determined by purely algebraic arguments, or by following a probabilistic approach. I shall focus on quasi-birth-and-death processes (QBDs), that is, Markov chains on a strip in the two-dimensional state space $\mathbb{N}^2$, and I shall show how one may exploit the special transition structure of QBDs, and the physical interpretation of the deviation matrix, in order to obtain an explicit expression in terms of easily obtained quantities.

My presentation is based on joint work with D. Bini, S. Dendievel, Y. Liu and B. Meini [1, 2]

## Keywords

Poisson equation, QBD process, Group inverse, Deviation matrix, Matrix difference equation.

## References

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# Optimal circular neighbor designs under mixed interference models

**Katarzyna Filipiak[1] and Augustyn Markiewicz[2]**

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[2] Poznań University of Life Sciences, Poland

## Abstract

The concept of neighbor designs was introduced and defined in [1] where some methods of their construction were also given. Henceforth many methods of construction of neighbor designs as well as of their generalizations are available in the literature. Recently, some results on optimality of specified neighbor designs under various linear models were obtained; cf. [2], [3]. The aim of the talk is to study the problem of optimality of circular neighbor designs under mixed model. It will include some recent result published in [4] as well as some new results. The study of optimality of designs under mixed model is based on the method presented in [5].

## Keywords

Neighbor designs, Circular balanced design, Universal optimality, Mixed model.

## References

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# Defining positive definite arithmetical functions and a partial order on the set of arithmetical functions by using matrix inequalities

**Mika Mattila**

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## Abstract

In this presentation we make use of the Löwner order on square matrices and induce a partial order on the set

$$\mathcal{A} = \{f : \mathbb{Z}^+ \to \mathbb{R}\}$$

of real-valued arithmetical functions. If $f$ and $g$ are given arithmetical functions, we define that $f \preceq g$ if and only if $(S)_f \preceq (S)_g$ for all $S = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{Z}^+$ and all $n = 1, 2, \ldots$, where $(S)_f = [f(\gcd(x_i, x_j))]$ and $(S)_g = [g(\gcd(x_i, x_j))]$ are the GCD matrices of the set $S$ with respect to function $f$ and $g$, respectively.
Positive definiteness of a function $f : \mathbb{R} \to \mathbb{C}$ is usually defined by demanding that the matrix $[f(x_i - x_j)] \in M_n$ is positive semidefinite for all choices of points $\{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}$ and all $n = 1, 2, \ldots$ [1, p. 400]. However, this definition does not work for arithmetical functions defined only on positive integers. By using our newly defined partial order it is natural to define that an arithmetical function $f$ is positive definite if and only if $f \succeq \mathbf{0}$, where $\mathbf{0}$ is the constant function having all of its values equal to 0.
We shall study the basic properties of our partial order $\preceq$ on $\mathcal{A}$ as well as properties of positive definite arithmetical functions. We also consider some elementary examples.

## Keywords

Arithmetical function, Positive definite function, Partial order, Löwner order, GCD matrix.

## References

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# Normal approximations to noncentral Wishart matrices

**Célia Nunes[1], Sandra S. Ferreira[1], Dário Ferreira[1], Manuela M. Oliveira[2], and João T. Mexia[3]**

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[2] Universidade de Évora, Portugal
[3] Universidade Nova de Lisboa, Portugal

## Abstract

The normal approximations we present hold when the norm of the non centrality parameters diverges to $+\infty$. Thus we have an attraction to the normal model, not for increasing predominance of the mean vectors over the constant variance-covariance matrices.

## Keywords

Asymptotic linearity, Limit normal distributions, Noncentral Wishart distributions.

## Acknowledgements

This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID/MAT/00297/2013 and UID/MAT/00212/2013.

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# Small Area Estimation for multivariate repeated measures data

**Innocent Ngaruye[1,2], Dietrich von Rosen[1,3], and Martin Singull[1]**

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[2] University of Rwanda, Rwanda
[3] Swedish University of Agricultural Sciences, Uppsala, Sweden

## Abstract

Small Area Estimation under a multivariate linear model for repeated measures data is considered. The aim of the proposed model is to get a model which borrows strength both across small areas and over time, by incorporating simultaneously the area effects and time correlation. The model accounts for repeated surveys, grouped response units and random effects variations. Estimation of model parameters is discussed within a restricted maximum likelihood based approach. Prediction of random effects and the prediction of small area means across time points and per group units are derived. An empirical study about crop yield estimation at district level for agricultural seasons 2014 in Rwanda is conducted.

## Keywords

Maximum likelihood, Multivariate linear model, Prediction of random effects, Repeated measures data, Small Area Estimation.

# Analysis of multivariate growth curves with smoothing splines

**Tapio Nummi**[1] **and Jyrki Möttönen**[2]

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[2] University of Helsinki, Finland

## Abstract

In this paper we investigate fitting and testing of multivariate growth curves when the analysis is based on smooth spline functions. It is shown that estimation is greatly simplified under certain important class of covariance structures. It is also shown how the approximated splines can be tested using the F-test. A real data example is used to illustrate the proposed methodology.

## Keywords

Complete and balanced data, Matrix normal distribution, Penalized likelihood.

# Comparing for one-way fixed effects models the usual and the random sample sizes ANOVA

**Célia Nunes[1], Dário Ferreira[1], Sandra S. Ferreira[1], and João T. Mexia[2]**

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[2] Universidade Nova de Lisboa, Portugal

## Abstract

We extend the theory of one-way fixed effects ANOVA to situations where the samples sizes may not be previously known. This often occurs when there is a fixed time span for collecting the observations. A motivation example is the collection of data from patients with several pathologies arriving at a hospital during a fixed time period, see e.g. [1, 2].
In these cases it is more appropriate to consider the sample sizes as realizations of independent random variables. We assume that the samples were generated by Poisson counting processes.
We present the test statistics and their conditional and unconditional distributions, under the assumption that we have random sample sizes. We also show how to compute correct critical values which may be important to avoid working with incorrect test levels, see [2].
Finally, we carry out with a simulation study, to compare and relate the performance of the proposed approach with those of common ANOVA.

## Keywords

Random sample sizes, Counting processes, Correct critical values, Simulation study.

## Acknowledgements
This work was partially supported by national founds of FCT-Foundation for Science and Technology under UID/MAT/00212/2013 and UID/MAT/ 00297/2013.

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# Linear sufficiency in the partitioned linear model

**Simo Puntanen**

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## Abstract

A linear statistic $Fy$, where $F$ is an $f \times n$ matrix, is called linearly sufficient for estimable parametric function $K\beta$ under the model $M = \{y, X\beta, V\}$, if there exists a matrix $A$ such that $AFy$ is the BLUE for $K\beta$. In this talk we consider some particular aspects of the linear sufficiency in the partitioned linear model where $X = (X_1 : X_2)$ with $\beta$ being partitioned accordingly. Our considerations are based on the properties of relevant covariance matrices and their expressions via certain orthogonal projectors. The connection between the transformed model $M_t = \{Fy, FX\beta, FVF'\}$ and the concept of linear sufficiency will have a crucial role. Particular attention will be paid to the situation under which adding new regressors (in $X_2$) does not affect the linear sufficiency of $Fy$.
Thanks for helpful discussions go to Augustyn Markiewicz and Radosław Kala.

## Keywords

Best linear unbiased estimator, Generalized inverse, Linear model, Linear sufficiency, Orthogonal projector, Löwner ordering, Transformed linear model.

## References

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# Inequalities of zeros of classical orthogonal polynomials via Jacobi matrices

**Kenier Castillo[1] and Fernando R. Rafaeli[2]**

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[2] Universidade Federal de Uberlândia, Brasil

## Abstract

In this talk we present some historical facts on monotonicity of zeros of classical orthogonal polynomials. Some of main tools is the Hellmann-Feynman theorem which provides information about the behavior of the eigenvalues of the Jacobi matrices.

# Goodness-of-fit tests for semiparametric and parametric hypotheses based on the probability weighted empirical characteristic function

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[2] North-West University, Potchefstroom, South Africa

## Abstract

We investigate the finite-sample properties of certain procedures which employ the novel notion of the probability weighted empirical characteristic function. We consider testing for multivariate normality with independent observations, and testing for multivariate normality of random effects in mixed models. Along with the new tests alternative methods based on the ordinary empirical characteristic function as well as other more well known procedures are implemented for the purpose of comparison.

## Keywords

Characteristic function, Empirical characteristic function, Goodness-of-fit test, Mixed model, Multivariate normal distribution.

# On the extension of a balanced mixed model

**Carla Santos**[1,4], **Célia Nunes**[2], **Cristina Dias**[3,4], **and João T. Mexia**[4]

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[4] Universidade Nova de Lisboa, Portugal

## Abstract

A model with orthogonal block structure, OBS, is a mixed model whose variance-covariance matrix is a linear combinations of known pairwise orthogonal orthogonal projection matrices, that add up to the identity matrix. When the orthogonal projection matrix on the space spanned by the mean vector commutes with the variance-covariance matrix we have a special class of OBS, models with commutative orthogonal block structure, COBS. This commutativity condition of COBS is a necessary and sufficient condition for the least square estimators, LSE, to be best linear unbiased estimators, BLUE, whatever the variance components.
Using the algebraic structure of the models, based on commutative Jordan algebras, and B-matrices, we study the possibility of obtaining COBS from the extension of balanced mixed models.

## Keywords

B-matrices, Jordan algebra, Mixed models, Models with commutative orthogonal block structure.

## References

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# Robust estimation in meta-regression analysis

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## Abstract

Meta analysis provides a quantitative method for combining results from separate independent studies with the same problem and has been frequently used in different areas of scientific research. However existing estimation methods are sensitive to the presence of outliers in the data sets. In this paper we explore the robust estimation for the parameters in meta-regression, including the between-study variance and regression parameters. Huber's rho function is adopted to derive the formulae of robust maximum likelihood (ML) and restricted maximum likelihood (REML) estimation. The asymptotic properties of proposed robust estimators are established and the derivation allows the true parameter lying on the boundary of parameter space. Corresponding iterative algorithm of robust estimation is developed which is easy to implement in the software. The performance of the proposed methodology is assessed by Monte Carlo simulation studies, and our results show that the robust estimation methods outperform the conventional $ML$ and $REML$ methods when outliers appear in the data set. Two real examples are used for illustrations.

# Method of divide-and-combine in regularized regression for Big Data

**Lu Tang, Ling Zhou, and Peter X.K. Song**

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## Abstract

When a data set is too big to be analyzed entirely once by one computer, the strategy of divide-and-combine (MODAC) has been the method of choice to overcome the computational hurdle. Although random data partition has been widely adopted, there is lack of clear theoretical justification and practical guidelines to combine results obtained from separately analyzed subdatasets, especially when a regularization method such as LASSO [2] is utilized for variable selection in the generalized linear model regression. We develop a new strategy to combine separately regularized estimates of regression parameters by means of the confidence distributions [3] of biased corrected estimators. We first establish the theory for the construction of the confidence distribution and then show that the resulting MODAC estimator enjoys the Fishers efficiency, the efficiency of the maximum likelihood estimator obtained from the analysis of entire data once. Furthermore, using the MODAC estimator we propose a variable selection procedure, which is compared analytically and numerically via extensive simulations with the existing majority-voting method [1] and the gold standard of one-time entire data analysis.

## Keywords

Confidence distribution, Generalized linear model, LASSO, Meta analysis.

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# A sensational 7 × 7 pandiagonal magic square with non-consecutive entries and "diamond-square arrangement" matrices for knight's tours in a pandiagonal magic carpet

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## Abstract

This "Sensational Pandiagonal Fully-Magic Matrix" with non-consecutive entries

$$\mathbf{A}_0 = \begin{pmatrix} 1 & 58 & 60 & 30 & 21 & 2 & 22 \\ 52 & 15 & 16 & 14 & 26 & 25 & 46 \\ 48 & 4 & 31 & 50 & 7 & 35 & 19 \\ 24 & 8 & 3 & 74 & 36 & 32 & 17 \\ 40 & 56 & 27 & 12 & 11 & 10 & 38 \\ 23 & 33 & 44 & 9 & 39 & 28 & 18 \\ 6 & 20 & 13 & 5 & 54 & 62 & 34 \end{pmatrix}$$

was created by Walter Trump (January 2016) so that we may build this philatelic "Machin magic square" based on $\frac{1}{2}\mathbf{A}_0$

$A_1$ $A_2$ $A_3$

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 58 | 60 | 30 | 21 | 2 | 22 | 1 | 58 | 60 | 30 | 21 | 2 | 22 | 1 | 58 | 60 | 30 | 21 | 2 | 22 |
| 52 | 15 | 16 | 14 | 26 | 25 | 46 | 52 | 15 | 16 | 14 | 26 | 25 | 46 | 52 | 15 | 16 | 14 | 26 | 25 | 46 |
| 48 | 4 | 31 | 50 | 7 | 35 | 19 | 48 | 4 | 31 | 50 | 7 | 35 | 19 | 48 | 4 | 31 | 50 | 7 | 35 | 19 |
| 24 | 8 | 3 | 74 | 36 | 32 | 17 | 24 | 8 | 3 | 74 | 36 | 32 | 17 | 24 | 8 | 3 | 74 | 36 | 32 | 17 |
| 40 | 56 | 27 | 12 | 11 | 10 | 38 | 40 | 56 | 27 | 12 | 11 | 10 | 38 | 40 | 56 | 27 | 12 | 11 | 10 | 38 |
| 23 | 33 | 44 | 9 | 39 | 28 | 18 | 23 | 33 | 44 | 9 | 39 | 28 | 18 | 23 | 33 | 44 | 9 | 39 | 28 | 18 |
| 6 | 20 | 13 | 5 | 54 | 62 | 34 | 6 | 20 | 13 | 5 | 54 | 62 | 34 | 6 | 20 | 13 | 5 | 54 | 62 | 34 |

In the $7 \times 21$ pandiagonal magic carpet above, based on our $7 \times 7$ "Sensational Pandiagonal Fully-Magic Matrix" $\mathbf{A}_0$, we find that for "Diamond-Square Arrangement" (DSA) $A_1$ in the left-side $5 \times 5$ sub-panel, the entries in the 4 cells in the outer-blue square have the same sum as the entries in the 4 cells in the inner-green diamond. For the DSA $A_2$ in the centre $7 \times 7$ panel ($\mathbf{A}_0$), the entries in the 4 cells in the outer-brown diamond have the same sum as the entries in the 4 cells in the inner-yellow square. For the DSA $A_3$ in the right-side $7 \times 7$ panel, the entries in the 4 cells in the outer-green square have the same sum as the entries in the 4 cells in the inner-blue diamond.

The 4 blue and the 4 green cells in the DSA $A_1$ in the left $5 \times 5$ sub-panel define a "knight's tour (CSP-2)" as do the 4 brown and the 4 yellow cells in DSA $A_2$ in the centre $7 \times 7$ panel. The 4 green and the 4 blue cells in DSA $A_3$ in the right-side $7 \times 7$ panel, however, define a "jumping rukh" tour or "special knight's tour (CSP-3)" as discussed by Styan in IWMS-2011 ["An illustrated introduction to Caïssan squares: the magic of chess", *Acta et Commentationes Universitatis Tartuensis de Mathematica,* vol. 16, no. 1, pp. 97–143 (2012).]

Let $\mathbf{A}$ be a $7 \times 7$ fully-magic matrix. We define the following $7 \times 7$ DSA matrices:

$$\mathbf{A}_1^{(\mathrm{dmd})} = \mathbf{A}\mathbf{X}_1 + \mathbf{X}_1\mathbf{A}, \quad \mathbf{A}_1^{(\mathrm{sqr})} = \mathbf{X}_2\mathbf{A}\mathbf{X}_2$$

$$\mathbf{A}_2^{(\mathrm{dmd})} = \mathbf{A}\mathbf{X}_3 + \mathbf{X}_3\mathbf{A}, \quad \mathbf{A}_2^{(\mathrm{sqr})} = \mathbf{X}_1\mathbf{A}\mathbf{X}_1$$

$$\mathbf{A}_3^{(\mathrm{dmd})} = \mathbf{A}\mathbf{X}_2 + \mathbf{X}_2\mathbf{A}, \quad \mathbf{A}_3^{(\mathrm{sqr})} = \mathbf{X}_3\mathbf{A}\mathbf{X}_3$$

where

$$\mathbf{X}_h = \mathbf{S}^h + \mathbf{S}^{-h} = \mathbf{S}^h + \mathbf{S}^{7-h}; \qquad h = 1, 2, 3$$

and $\mathbf{S}$ is the "1-step-forward shift matrix" $\begin{pmatrix} 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0 \end{pmatrix}$. Then we conjecture that

$$\begin{aligned} \mathbf{A} \text{ is pandiagonal} \quad &\Leftrightarrow \quad \mathbf{A}_1^{(\mathrm{dmd})} = \mathbf{A}_1^{(\mathrm{sqr})} \quad \Leftrightarrow \quad \mathbf{A}_2^{(\mathrm{dmd})} = \mathbf{A}_2^{(\mathrm{sqr})} \\ &\Leftrightarrow \quad \mathbf{A}_3^{(\mathrm{dmd})} = \mathbf{A}_3^{(\mathrm{sqr})}. \end{aligned}$$

# Some comments on Sophie Germain prime numbers and on two philatelic magic-carpet dinner-placemats for the IWMS-2016 Madeira Magic Minisymposium

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## Abstract

The prime number $p$ is a "Sophie Germain prime" if $2p + 1$ is also prime. The number $2p + 1$ associated with a Sophie Germain prime is called a safe prime. For example, 29 is a "Sophie Germain prime" and $2 \times 29 + 1 = 59$ is its "associated safe prime".


Sophie Germain primes are named after the French mathematician Marie-Sophie Germain (1776–1831), who used them in her investigations of Fermat's Last Theorem. Sophie Germain primes and safe primes have applications in public key cryptography. In 1794, when Sophie Germain was 18, the École Polytechnique opened. As a woman, Sophie Germain was barred from attending, but the new system of education made the "lecture notes available to all who asked". Germain obtained the lecture notes and began sending her work to the mathematician and astronomer Joseph-Louis Lagrange (1736–1813). Sophie Germain primes less than 1000 are (OEIS A005384):

$$2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, 131, 173, 179, 191, 233, 239, 251, 281, 293,$$
$$359, 419, 431, 443, 491, 509, 593, 641, 653, 659, 683, 719, 743, 761, 809, 911, 953.$$

The 4 largest Sophie Germain primes that we have found as face values on individual stamps are 239 (Andorra), 293 (Curaçao), 359 (Malta), 593 (France).


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Philatelic magic-carpet dinner-placemat for the IWMS-2016 Madeira Magic Minisymposium


# The Markov-modulated Erlang Loss System

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[1] University of Amsterdam, Netherlands
[2] University of Melbourne, Australia
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## Abstract

Since it was first proposed by Erlang in 1917, the *Erlang Loss model* has arguably been the most successful contribution by queueing theory to the dimensioning of telecommunication systems. In this talk we shall discuss a generalisation of this model loss in which both the arrival rate and the per-customer service rate vary according to the state of an underlying finite-state, continuous-time Markov chain. We can think of such a system as a Markov-modulated version of the Erlang Loss model.
We obtain a closed-form matrix expression for the stationary distribution of this queue. This, in particular, provides us with an explicit expression for the stationary probability that the queue is full, which can be regarded as the Markov-modulated counterpart of the famous Erlang loss formula.We can use this expression to compute a number of performance measures of interest, in particular the the probability that an arbitrary arriving customer is blocked.

## Keywords

Erlang Loss System, Markov Modulated process.

# Modeling the caregivers knowledge about pediatric hypertension

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[3] Centro Hospitalar Lisboa Norte, Portugal

## Abstract

The high arterial blood pressure is a condition which, although traditionally considered a disease of adults, may increase during the pediatric age and in most cases, silently. The diagnostic criteria for Pediatric Hypertension (HT) have as their main reference the normal distribution of blood pressure (BP) in healthy children [1] and based on the concept that the pediatric BP increases with age and with body mass [3].
To evaluate the population's knowledge about the disease, it was applied an experimental questionnaire to pediatric patients caregivers of Hospital of Santa Maria.
Some statistical analysis using partial data of such questionnaire can be found in [4]. The use of some additional statistical methods, similarly to [2], give us another an extra contribution to the data variability explanation.
In the present article, we extend the work presented in [4], where the statistical approach includes the use of statistical methods such as correspondence analysis or other multivariate methods adequate to analyze the kind of data under study. The results are promising but some details still need to be completed.

## Keywords

Hypertension, Children knowledge, Caregivers, Multivariate methods, Generalized linear models.

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# A new algebraic analysis of linear mixed models

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## Abstract

This article presents a new investigation to the linear mixed model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}$ with fixed effect $\mathbf{X}\boldsymbol{\beta}$ and random effect $\mathbf{Z}\boldsymbol{\gamma}$ under a general assumption via some novel algebraic tools in matrix theory, and reveals a variety of deep and profound properties hidden behind the linear mixed model. We first derive exact formulas for calculating the best linear unbiased predictor (BLUP) of a general vector $\boldsymbol{\phi} = \mathbf{F}\boldsymbol{\beta} + \mathbf{G}\boldsymbol{\gamma} + \mathbf{H}\boldsymbol{\varepsilon}$ of all unknown parameters in the model by solving a constrained quadratic matrix-valued function optimization problem in the Löwner partial ordering. We then consider some special cases of the BLUP for different choices of $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ in $\boldsymbol{\phi}$, and establish some fundamental decomposition equalities for the observed random vector $\mathbf{y}$ and its covariance matrix.

## Keywords

Linear mixed model, Fixed effect, Random effect, BLUP, BLUE, Covariance matrix, Decomposition.

# Clusterwise regression using mixtures of regression models

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## Abstract

The classical linear regression approach is reasonable under the assumption of a homogeneous population. Nevertheless, when we suspect that there are several heterogeneous groups in the population represented by a given sample, then mixtures of regression models can be applied. The method acquires parameters estimates by modelling the mixture conditional distribution of the response given the explanatory variable.
Provided the mixture consists of $c$ components, the mixture distribution is given by the weighted sum over all $c$ components. Parameters of a mixture of linear regression models are estimated by maximum likelihood using the expectation maximization (EM) algorithm. In order to improve regression parameter estimates and data classification, mixture model can be extended to include concomitant variables. These additional variables influence the weights of a mixture regression model so they are no longer deterministic but they operate as functions of one or more concomitant variables.
Recently, mixture models are used more and more in a various fields, including the economics. The methodology will be illustrated on an analyses of the relationship between an old age pension and income of EU countries residents older than 65 years.

## Keywords

Cluster analysis, Mixture regression models, Linear regression, EM algorithm, Old age pension, Income.

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# Comparison of estimators in a multivariate linear model with generalized uniform correlation structure

**Ivan Žežula**

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## Abstract

We consider multivariate linear model in the form

$$Y_{n\times p} = X_{n\times m}B_{m\times p} + \varepsilon_{n\times p}, \quad E\varepsilon = 0, \quad \operatorname{var}\operatorname{vec}(\varepsilon) = \varSigma_{p\times p} \otimes I_n,$$

where $\varSigma = \theta_1 G + \theta_2 ww'$, $G \geq 0$, $w \in \mathcal{R}(G)$.
We study and compare properties of three different proposed estimators of the variance parameters $\theta_1$ and $\theta_2$ (see [1], [3], [2]).

## Keywords

Multivariate linear model, Generalized uniform correlation structure, Intraclass correlation structure.

## Acknowledgements

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# Application of Jordan algebra for statistical inference in multivariate normal models

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[2] University of Zielona Góra, Poland

## Abstract

It will be presented applications of Jordan Algebra to the problem of optimal estimation and testing hypotheses in multivariate normally distributed models. As an example are block compound symmetric (BCS) covariance structure in multivariate models. The test are based on quadratic unbiased estimators of covariance parameters as a ratio of positive and negative part of estimator of covariance parameters.

## Keywords

Sufficient and complete statistics, Unbiased estimation, Testing hypotheses, Block compound symmetric covariance structure.

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# Part VI

# Posters

# Unbiased estimator using hypergeometric function

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## Abstract

In the Growth curve model uniform correlation structure is of the interest. Most estimators of parameter $\rho$ are biased. Olkin and Pratt in 1958 [2] showed how to derive unbiased estimator of correlation coefficient using hypergeometric function. Our motivation for studying unbiased estimator is Ye, Wang in 2009 [3].
Our aim is to derive unbiased estimator of $\rho$ in case of uniform correlation structure based on moment estimator derived by Klein and Žežula in 2010 [1] and compare these estimators.

## Keywords

Growth curve model, Unbiased estimator, Hypergeometric function.

## Acknowledgements

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# Hybrid model for recurrent event data

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## Abstract

In the last four decades, there has been an increasing interest in developing survival models appropriate for multiple event data and, in particular, for recurrent event data. Some of the most known models for the last situation are PWP (Prentice, Williams and Peterson) [5], AG (Andersen and Gill) [1], WLW (Wei, Lin and Weissfeld) [6] and LWA (Lee, Wei and Amato) [4], all of them extensions of the Cox model [3]. These models can handle with situations where exists potentially correlated lifetimes of the same subject (due to the occurrence of more than one event for each subject) which is common in this type of data.
In this work we will present a new model, which we will call hybrid model, with the purpose of minimizing one of the limitations of PWP model: the violation of the missing completely at random (MCAR) condition [2].

## Keywords

Correlated observations, Cox model, Recurrent events, Survival analysis.

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# Part VII

## Jeffrey Joseph Hunter

# Short Biography of Jeffrey J. Hunter

Professor of Mathematical Sciences, Auckland University of Technology,
Auckland, New Zealand
Professor Emeritus of Statistics, Massey University, New Zealand

Jeffrey Hunter is a New Zealander, educated at King's College, Auckland, who graduated in 1962 with a B.Sc. degree and, in 1963, an M.Sc. degree, with First Class Honours in Mathematics from the University of Auckland. He accepted a Fulbright grant to attend the University of North Carolina at Chapel Hill, U.S.A., where he completed a Ph.D. degree in Statistics in 1968. He returned to a Lectureship at the University of Auckland rising to the rank of an Associate Professor in Statistics. In 1990 he accepted the Chair in Statistics at Massey University, Palmerston North Campus within the Department of Mathematics and Statistics. In 1991 he became the Foundation Head of the Department of Statistics and in 1995 became the first permanent Dean of the newly established Faculty of Information and Mathematical Sciences. Following the formation of College of Sciences in Massey University in 1998, he moved back to Auckland to become the Foundation Head of the Institute of Information and Mathematical Sciences on Massey University's newly established Albany Campus. He stepped aside from this role in 2001 to return to the position of Professor of Statistics. He was granted Professor Emeritus status on his retirement from Massey University in November 2007. He then joined Auckland University of Technology as Professor of Mathematical Sciences. He is currently Head of Research in the Department of Mathematical Sciences, having been Head of Mathematical Sciences, in the School of Computer and Mathematical Sciences, now a separate department in the renamed School of Engineering, Computer & Mathematical Sciences.

During his career he has held visiting positions at the Department of Statistics, University of North Carolina at Chapel Hill, (1973, 1988, 2001), Department of Industrial Engineering and Operations Research, Virginia Polytechnic Institute and State University, Blacksburg, (College of Engineering Visiting Professor 1980, 1987) and has been an academic visitor at the Department of Mathematics and Statistics, McGill University, Montreal, (1988), Mathematical Sciences Institute, Cornell University (1988), Statistical Laboratory, University of Cambridge, U.K. (1992), Department of Industrial Engineering, Texas A & M University (1992), Department of Statistics, University of Oxford, U.K., (2002), and the Hamilton Institute, National University of Ireland (2012).

In 1992 he was awarded a Claude McCarthy Fellowship to visit universities in the U.S., Canada, United Kingdom and South Africa. In 1992 he was the Distinguished Lecturer in Applied Probability, Department of Industrial Engineering, Texas A & M University. In 1993 he was elected as a Member of the International Statistical Institute.

Professor Hunter has been a member of the New Zealand Mathematical, Statistical and Operations Research Societies for the duration of his career. He was President of the New Zealand Statistical Association (1995–97), elected a Fellow of the New Zealand Mathematical Society (2002), and served as Chair of the Royal Society of New Zealand Committee on Mathematical and Information Sciences (1997–2002). For his services to the Mathematical Sciences he was awarded a Bronze New Zealand Science & Technology Medal in 2003. In 2006 he was the recipient of the Campbell Award, the highest award given by the New Zealand Statistical Association "to recognize his contributions to statistical research and education, and his services to the profession of statistics". He was also given Honorary Life Membership of the NZSA. In 2008 he was made a Rotary International Paul Harris Fellow as member of the Rotary Club of Auckland East. In 2015 he was appointed to the Advisory Board of Christ's University in Pacific, Tonga.

Professor Hunter's research has been in Applied Probability and Linear Algebra focusing on Markov chains, semi-Markov processes, generalized matrix inverses, queueing theory and two-dimensional renewal theory. In 1983 Academic Press published his two-volume work on "Mathematical Techniques in Applied Probability". In 2005 he graduated with a D.Sc. degree in Applied Probability from Massey University based on his published research in this field.

Professor Hunter has been involved with organisation of a number of national and international conferences including Chair of the Local Organising Committee of the International Workshop on Matrices and Statistics held at Massey University, Albany Campus 2005. He joined the International Organising Committee in 2006 and has been the Chair of the IOC for the workshops at Shanghai in 2010 and Hainan in 2015. He has also been an invited speaker at a number of international conferences in Australia, Canada, Czech Republic, Greece, India, Israel, Italy, Netherlands, New Zealand, New Caledonia, Poland, Portugal, South Korea, Spain, and Sweden.

# Publications

**Jeffrey Joseph Hunter** B.Sc., (NZ), M.Sc. (Hons), (Auckland), Ph.D. (North Carolina), D.Sc. (Massey), MRSNZ, FNZMS

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# Many happy returns, Jeffrey Joseph Hunter! Personal Glimpses for your 75th Birthday

**Simo Puntanen**

University of Tampere, Finland

Jeffrey Joseph Hunter was born 75 years ago, 27 March 1941; Otahuhu, in a suburb of Auckland, New Zealand. His father was a local businessman (Hunter's Radio Ltd) and Jeff was given the opportunity to be educated at King's College over the years 1954–58. According to Wikipedia, Otahuhu was home to the country's first supermarket, and King's College is one of the largest boarding schools in New Zealand, being originally a boys-only school but now also admits girls in the sixth and seventh forms (years 12 and 13).

I first met Jeff in May 1985 in Auckland, and by then, Jeff was pretty fully-educated and after all the wild years was enjoying regular family life with his wife Hazel and children Michelle and Mark. The key to my first visit to New Zealand was my collaborator George P. H. Styan who was spending his sabbatical 1984–85 in Auckland. The University of Auckland organized the First Pacific Statistical Congress, 20–24 May 1985, *Proceedings* published in 1986. The Photograph 2 is taken at the reception of this conference. One evening during the conference time, Hazel and Jeff organized a cozy dinner party at their home in St Heliers, see Photograph 1. I still remember how heavy the rain (153 mm in one 24-hour interval) was that evening – when it rains in New Zealand it really rains.

The month of May 1985 was a starting point for my long-lasting delightful co-operation with the Kiwi friends: how could I have imagined that 25 years later on 20th of May 2010, my wife Soile and I would arrive at 6:55 AM by AY51 at the Beijing Capital Airport and meet 20 Kiwis arriving from Auckland at 7:20 AM by NZ87 and then hit the road for two weeks in China under the excellent leadership of Jeff! To Soile and me it all nicely resulted Honorary Citizenships of New Zealand.

Let me add one memorable thing: upon our arrival at Beijing, we had to kill some time before we could accommodate our exhausted bodies to our hotel and for that exercise Jeff had arranged us a visit to the Temple of Heaven. Believe me, it was a hot day. In the same scale hot as the lecture room in Ljubljana's International Workshops on Matrices and Statistics (IWMS) in June 2014 and the weather in Hainan Island in May 2015 – if you know what I mean . . . and I'm not joking.

Returning back to George I would like to cite Jeff (2012) who in his article entitled "George P. H. Styan. A celebration of 75 years: A personal tribute" described his history with George as follows:

PHOTO **1.** Margaret Scott, Ross Leadbetter, and Jeff at Château Hunter, St Heliers, Auckland, May 1985.


PHOTO **2.** Jeff discussing with Ivor S. Francis; George P. H. Styan evaluating the wine, Chris Triggs pouring the wine, Terry Speed in the horizon; Auckland, May 1985.

"My association with George goes back to 1973 when I attended an Institute of Mathematical Statistics meeting at Ithaca College to hear George talk on some research on Markov chains that included reference to generalized inverses. This was of much interest to me as I had published a paper in 1969 identifying Kemeny and Snell's fundamental matrix of Markov chains as a generalized inverse. Starting from that meeting our subsequent association has spanned the globe

Photo **3.** St Heliers again, 17 April 2005. Clockwise: Hazel, Jarkko Isotalo (hardly visible), Alastair Scott, Soile P., Jeff, George P. H. Styan, Margaret Scott.

> with George visiting the University of Auckland over the period July 1984 to June 1985 on a sabbatical leave to spend time primarily with George Seber and Alastair Scott. At that time I was a member of the Department of Mathematics and Statistics at the University of Auckland. I followed up his visit with me visiting McGill University for a month in May 1988 and again visiting McGill in June 2001 (when I was based at Massey University). Both of these latter visits occurred while I was on a sabbatical leave. George tried to discourage me from pursuing any further activity on generalised inverses but not all was known about their properties when associated with Markov chains so that I failed to take his advice!"

While we are now in old good times, let's go back a bit further. Ingram Olkin, the famous Stanford Professor, was coming, as a Keynote Speaker, to the IWMS meeting in Ljubljana, June 2014. While communicating with Ingram, Jeff was refreshing his memories from the University of North Carolina at Chapel Hill, where also Ingram had completed his dissertation (in 1951). Jeff cc'ed his email to me on 17 April 2014, and (with his kind permission) I copy some interesting reminiscences here:

> "[writing to Ingram Olkin: . . . ] I did enjoy reading your comments about the North Carolina statisticians as I also received my PhD from UNC-Chapel Hill. I remember Hotelling (and in fact was given his personal copies of the Kendall and Stuart volumes at the time of his death when I was revisiting UNC in 1973 on a sabbatical leave). I also remember Alfred Brauer – he was a quiet charming man. R. C. Bose was still in the Department – I was at Chapel Hill 1964-68 – and I have a copy of the mimeographed notes that Bose prepared (they were used by Indra Chakravarti) where he introduced 'conditional

PHOTO **4.** Excursion to Waiheke Island, Auckland, 30 March 2005.

PHOTO **5.** Jeff and C. R. Rao; Auckland, 31 March 2005. (Eugene Seneta with a camera.) Ph: Harold V. Henderson.

PHOTO **6.** Bhargavi and C. R. Rao, George P. H. Styan, Hans Joachim Werner, Shayle R. Searle; Auckland, 1 April 2005. Ph: Jeffrey J. Hunter.

inverses' in solving systems of linear equations before considering linear estimation with fixed effects and tests of linear hypotheses. It was in the course on 'Analysis of Variance with Applications to Experimental Design'. I also had a course from Wassily Hoeffding and I learnt my Statistical Theory in two demanding courses from Norman Lloyd Johnson. It was a great Department to study and work in. After my degree I stayed on for nine months as a Research Associate

PHOTO **7.** Jeff celebrating the winning of the beer tasting competition in the IWMS, Dortmund, July 2003.

PHOTO **8.** Jeff enjoying lobsters, conference banquet, SSC Meeting in St. John's, Newfoundland, Canada, June 2007.

> and started applying my knowledge on what was then called 'generalised inverses' to problems in Markov chained and Markov renewal processes. I am still using those techniques today!"

As I mentioned earlier, in May 1985 I was hosted at the Château Hunter in St Heliers by Jeff and Hazel, but it's fair to mention that 20 years later, 17 April 2005, the same happened around the Auckland's IWMS meeting. Then I was happy not only to have Soile with me but also my PhD student Jarkko Isotalo. I still wonder if Jeff ever realized my utmost (though hopeless) eagerness to get the light flashing between Michelle and Jarkko: what a wonderful excuse to visit New Zealand (to check Jarkko's research progress) if something everlasting would have started . . .

Jeff is an NBL, a natural-born-leader. I have had a pleasure to experience this not only when touring with him, for example, in China and South India, but also following closely his chairing the organizing committees of the IWMS in

- Auckland, 29 March–1 April 2005, www, Report in *Image* (2005),
- Shanghai, 5–8 June 2010, www, Report in *Image* (2010),
- Haikou City, Hainan Island, China, 25–28 May 2015, www, *Souvenir Booklet* (2015c), Report in *Image* (2015b),

and being an active member of the organizing committees of several other IWMSs, like the second IWMS held 4–5 December 1992 in Auckland,

It was following Jeff's visit with George P. H. Styan in Montreal in 2001 that he was persuaded to chair the Local Organizing Committee of the 14th IWMS in Auckland in spring 2005. Jeff was able to get Professor C. Radhakrishna Rao to New Zealand not only as a Keynote Speaker and the Nokia Lecturer for the IWMS but also as a New Zealand Statistical Association Visiting Lecturer. As a result Professor Rao was touring and performing, like the Rolling Stones, in six universities throughout New Zealand in 14–23 March.

PHOTO **9.** Conference banquet: Shenfang Zheng, Hazel, Jeff, Guiqing Wang; IWMS-Shanghai, 6 June 2010.

PHOTO **10.** Hans Joachim Werner, Kai-Tai Fang, Yonghui Liu, Jeff, Shenfang Zheng, SP, Tingmui Li; IWMS-Shanghai, 6 June 2010. Photo: Soile P.

The 22nd IWMS was held in Toronto, at the Fields Institute for Mathematical Research, 12–15 August 2013. The highlight of the event was the Memorial Session and Reception to Honor Shayle Robert Searle (1928–2013) with speakers: David A. Harville, Jeffrey J. Hunter, Jon N.K. Rao, Robert Rodriguez, Susan Searle, and Heather Selvaggio. The participants in particular appreciated the wonderful personal reminiscences of Susan Searle and Heather Selvaggio, the two daughters of Shayle Searle. This Special Session was arranged and chaired by Jeff as well as summarised by him in the *Australian & New Zealand Journal of Statistics* (2015a): Shayle was Jeff's close personal friend, having met first time in 1968 they had a long-standing association over the years both in New Zealand and in Ithaca (Cornell University). Shayle also attended several IWMS meetings. In Tampere, in August 1999, we had a banquet in the Viikinsaari Island and part of the pre-dinner program was a penalty kick competition. When it was Shayle's turn, he took his walking stick, put it upside down and hit the ball. I was the referee and of course, I accepted the goal with pleasure. – Before leaving Shayle Searle, let me mention that the session in which I gave my talk in the First Pacific Statistical Congress, 20–24 May 1985, was chaired by Shayle. He was not a sleepy chair and did not give any extra minutes.

PHOTO **11.** Speakers at the Memorial Session in Honour of Shayle R. Searle, Fields Institute, Toronto, 14 August 2013. Jeffrey J. Hunter, Robert Rodriguez, Susan Searle (daughter, South Carolina), David A. Harville, Heather Selvaggio (daughter, Florida), Jon N. K. Rao. Ph: Joanna Modlawska.

On 25–28 May 2015 we had our IWMS meeting in Haikou, in Hainan Island, in South China. Jeff was the Chair of the International Organizing Committee and I can tell you (because I happen to know) that Jeff did an awful lot of work for the meeting! The success of the event was no doubt very much

Photo **12.** George Styan, Evelyn Styan, Hazel, Jeff, Mrs Andrews, Eileen Wild, David F. Andrews, Chris Wild, Soile P., Hans Joachim Werner, Magdala Werner; Annual Meeting of the Statistical Society of Canada (SSC), St. John's, Newfoundland, June 2007.

due to Jeff's devotion and tireless efforts. So impressive to follow closely his determined actions – believe me!

In addition to his impressive list of academic administrative tasks, Jeff is the author of numerous refereed articles in top-journals, author of books, editor of special issues of journals, and a teacher of statistics, probability and operations research. Jeff's research has been in applied probability, focusing on Markov and semi-Markov processes, generalized matrix inverses, queueing theory and two-dimensional renewal theory. He was the President of the New Zealand Statistical Association during 1995–97 (eventually serving on its Executive for 13 years, gaining honorary lifetime membership). All in all, he's a thoroughly experienced University Professor, familiar with all spices the academic life can offer.

In 2007 George P. H. Styan and Götz Trenkler interestingly and colourfully characterized Jeff's research topics in their paper "A philatelic excursion with Jeff Hunter in probability and matrix theory" in the Special Issue of the *Journal of Applied Mathematics and Decision Sciences*. Here is their abstract:

> "We present an excursion with Jeff Hunter, visiting some of his research topics. Specifically, we will present some facts about certain people whose work seems to have influenced Jeff in his scientific career; we illustrate our presentation with postage stamps that have been issued in honour of these people. Our main guide is Hunter's two-volume book (1983a; 1983b) entitled *Mathematical Techniques of Applied Probability* (Academic Press)."

PHOTO **13.** Hazel and Jeff in the night of Shanghai, 24 May 2010.

PHOTO **14.** Enjoying the IWMS banquet, Shanghai, 6 June 2010.

In April 2005, Jeff graduated with his Doctor of Science degree in Applied Probability from Massey University; his thesis (2005b) was a compilation of his research papers spanning his career. In 2006 Jeff received the Campbell Award, the highest award of the New Zealand Statistical Association, to recognise his contributions to statistical research and education, and his services to the profession of Statistics. In 2007 Jeff was appointed Professor Emeritus of Statistics, at Massey University. Since then he's been working at the Auckland University of Technology, School of Computing and Mathematical Sciences, where he is now currently Head of Research in the Department of Mathematical Sciences, having been Head of Mathematical Sciences, in the School of Computer and Mathematical Sciences, now a separate department in the renamed School of Engineering, Computer & Mathematical Sciences.

*But what happened before?*

Did I already mention what a wonderful swimmer Jeff has been (no doubt still is)? Well, I skip this for the time being and jump directly to the years of 1960s.

After receiving his Master's degree, with First Class Honours in Mathematics at the University of Auckland in 1963, Jeff stayed the years 1964–68 in the University of North Carolina at Chapel Hill, USA, receiving his Ph.D. in

Photo **15.** Jeff running a guided tour, Xian, China, May 2010.


Photo **16.** Seating order in the IWMS banquet, Shanghai, 6 June 2010.

Statistics in June 1968, with the thesis "On the Renewal Density Matrix of a Semi-Markov Process" (adviser Walter L. Smith). In 1969–90, Jeff worked at the University of Auckland, and in 1990–2007 at the Massey University, working at the Institute of Information and Mathematical Sciences (IIMS), Albany Campus, in 1998–2007.

It is interesting to cite here the Newsletter of the IIMS (2005b) where Jeff describes his research philosophy as follows:

> "Over the years I have kept a notebook sectionalized into various topics with a list of key papers that referred to that problem. This went with me wherever I went! In many instances as the result of my reading (and thinking!) new ideas started to emerge. This was the precursor to the next stage in my 'research process'. This entailed putting the background material away and putting my own ideas to paper. Stored in one's brain are concepts and approaches that others have taken but the important next stage is to really flesh out your own approach. Nothing is better that putting pen (or pencil!) to paper in a quiet environment and seeing what evolves! You are often surprised and once you have the problem 'tiger by its tail' it is difficult to put it aside. It often preoccupies your thoughts for many a day – the thought process is often ongoing – and the urge to continue grappling with the problem insatiable. Your partner often detects that you appear to be in a trance and non-communicative. The pleasure at solving the problem is however very satisfying."

PHOTO **17.** Hazel and Jeff in Skokloster Castle, Sweden, June 2006.

Jeff and Hazel are great travellers. However, as I now understand the situation (writing this in March 2016), Hazel unfortunately may not make it this time (June 2016) to Madeira. Anyways, I'm convinced that we'll have many opportunities to meet in the future, take care, Hazel!
I wish you all the very best for the many years to come, Jeff!

**Acknowledgements**

Thanks go to Jeffrey J. Hunter and Kimmo Vehkalahti for helpful comments, and for the photographs 5, 6, 11, and 20 to Harold V. Henderson, Jeffrey J. Hunter, Joanna Modlawska, and Yonghui Liu. respectively; the other photos, except #28, are taken by SP. Part of this article appeared in the *Birthday Recognition Book* of the *Probastat Conference*, Smolenice Castle, Slovakia, 4–8 July 2011, when celebrating Jeffrey J. Hunter's 70th birthday.

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## More Photographs


PHOTO **18.** In the banquet podium with Yonghui Liu, IWMS-Shanghai, 6 June 2010.


PHOTO **19.** Opening ceremonies of the IWMS-Haikou, 25 May 2015. Ph: Soile P.


PHOTO **20.** Steve Haslett, Jeff, Peter Taylor, Shanghai, June 2010. Ph: Yonghui Liu.

PHOTO **21.** Ranganathittu Bird Sanctuary, Karnataka, India, 9 January 2012. Excursion arranged when attending the International Workshop & Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices, Manipal University, Manipal, India, www.

Photo **22.** Jeff checking his camera in Manipal banquet, 10 January 2012; Jeff, Hazel, SP, Augustyn Markiewicz, Tonu Kōllo, Steve Kirkland.

Photo **23.** Jeff zooming in a peninsula, Pyhäjärvi Lake, Tampere, near Pyynikki Park, 24 June 2011.


Photo **24.** Jeff omitting an opportunity to buy a photograph of him with/without birds: only RMB 5! Li River Cruise Guilin–Yangshuo in China, 28 May 2010.

Photo **25.** Jeff and George having fun in the IWMS-banquet, Ljubljana, June 2014.


PHOTO **26.** Soile, Hazel and Jeff exploring Tomar, Portugal, 11 July 2011.

PHOTO **27.** Jeff and Hazel with Tingmui Li and Kai-Tai Fang, IWMS-Haikou, 25 May 2015.


PHOTO **28.** IWMS-2015, Haikou, 25 May 2015. Ph: Hainan Normal University.

# Part VIII

# Memories of Ingram Olkin

# Ingram Olkin (1924–2016): Some Personal Memories ⋆

**Simo Puntanen[1] and George P. H. Styan[2]**

[1] University of Tampere, Finland
[2] McGill University, Montréal (Québec), Canada

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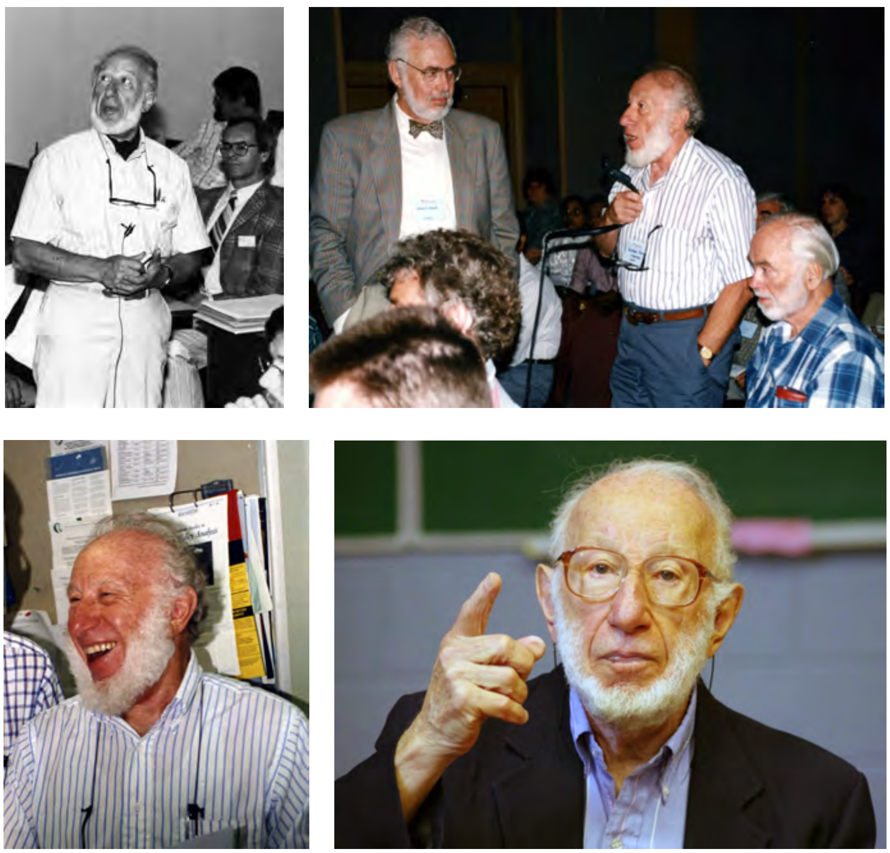

Top left $p_{11}$ at IWMS-1990 Tampere (with Jerzy K. Baksalary & Yadolah Dodge, photograph courtesy University of Tampere);
top right $p_{12}$ at IWMS-1995 Montréal (with Gene H. Golub and T. W. Anderson, photograph by Simo Puntanen); bottom left $p_{21}$ at IWMS-1995 Montréal (photograph by Simo Puntanen); bottom right $p_{22}$ at IWMS-2011 Tartu, Estonia (photograph by Jeffrey J. Hunter).

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⋆ A shortened version appeared in *Image: The Bulletin of the International Linear Algebra Society*, 56 (Spring 2016), pp. 22–26.

Ingram Olkin, Professor Emeritus of Statistics and Education at Stanford, University, Master of multivariate statistical analysis, linear algebra, inequalities, majorization, and meta-analysis, passed away on 28 April 2016 at home in Palo Alto, California, after complications from colon cancer. In the words of his daughter Julia Olkin [1]

> "My father, Ingram Olkin, died peacefully on Thursday evening, April 28, 2016, with his daughter Rhoda and wife Anita by his side. He had absolutely no regrets ... both personally and professionally, and led a full, wonderful life. He valued all his friendships with everyone. Thank you for being a part of his life ..."

Richard W. Cottle, Professor Emeritus of Management Science & Engineering and a close friend of Olkin, said [2]

> "He was a man of remarkable intelligence and affability. His nearly boundless energy was generously used for the welfare of others. It is hard to capture in words the goodness that Ingram showed in his everyday life."

In the conversation part of the *Olkin Festschrift* [3], Ingram described himself:

> "You also know that I'm generally a people person, which is one of the reasons why I've enjoyed students and collaborators. Over the years, the professional contacts have merged with the personal contacts."

We deeply miss you, a truly outstanding and unforgettable *People Person*, Ingram Olkin.

## 1 IWMS

Now let's go back to some personal memories of Ingram and joint experiences that we shared with him. One important activity for us was his role in the International Workshop on Matrices and Statistics (IWMS) series [4]. Ingram was a frequent participant at IWMS meetings, and at the IWMS-2004 in Poland we celebrated Ingram's 80th Birthday. On 4 June 2003 his reply to our invitation was this e-mail [Ingram usually used only lower-case letters in his e-mails]

> "dear all ... wow !!!! how about celebrating my 80th but call it my 60th ... thanks so much to all of you ... would be pleased to attend."

When Ingram learnt that the IWMS-2014 was to be held in Ljubljana, he immediately, on 22 October 2013, sent this e-mail

> " ...in any case next year is my 90th and what better than to visit ljubljana ...so i do hope to attend. as i see my strength at this point i should be in good shape by then. so please include me in the program."

It was always great news for the event organizers to have Ingram around: a guarantee of lively colorful sessions, Ingram sitting in the front row and asking questions after each talk. Ingram's role in meetings is nicely described in the Olkin-biography article [6]:

> "At most statistics meetings, you will find Ingram in constant conversation – perhaps promoting a new journal, encouraging progress of a key committee, or giving advice about seeking grants or allocating funds. His public accomplishments are many and impressive, but equally important are his behind-the-scenes contributions."

The first IWMS was held in Tampere, Finland, 6–8 August 1990. Ingram gave an invited talk entitled *Interface between statistics and linear algebra,* which was one of his favorite topics and he practically knew everything about it [9, 10]. For the IWMS-2013 in Toronto he prepared an excellent "linear algebra biography", which was presented there as a poster; see also [11] (2015):

> "I gave a brief biography of my introduction to linear algebra and my interaction with some of the linear algebraists at that time."

At the IWMS-1990 in Tampere, Ingram also gave a talk about Gustav Elfving (1908–1984), a famous Finnish statistician, probabilist and mathematician who was a frequent visitor to Stanford. On 19 March 2013 Ingram sent this e-mail to Simo:

> "i am cleaning my files and i found folder marked elfving which contains mimeographed noted entitled bayes statistics. it consists of about 40 pages ... so one possibility is that i scan these and send to you ... assuming you want this material ... please advise."

As for Elfving, on 20 May 2011 Ingram wrote the following:

> "... my only concern is how to handle the mixture of beer and aquavit. I don't have the right DNA. I once visited gustav elfving and he took me to a meeting of students where they drank beer and aquavit and talked and drank and sang and drank ... i barely made it back to the hotel. So Finland can be a very dangerous country ... but i am willing to take a chance."

Ingram's performances in Tampere in 1990 can be seen in videos online at YouTube [16]. When we asked for Ingram's permission to show these videos, he replied:

> "these are wonderful . . . an absolutely great addition to the conference archives. however, you ask for me to give permission to make these public. the answer is in the negative unless you can add some hair and make me look more like james bond. of course, if you do that then i would be glad to grant permission !!!!!"

This e-mail is part of the communication between Ingram, Michael Greenacre and Kimmo Vehkalahti. Kimmo had agreed to host Ingram and Michael Greenacre in Helsinki, 1–3 July 2011, directly after the IWMS-2011 in Tartu, Estonia. With the kind courtesy of Kimmo, we copy here part of Ingram's travelling protocol.

> "dear kimmo: on the basis of my previous experiences in finland I suggest that we just go to a Sauna, drink some beer and listen to michael [Greenacre] sing some of his compositions. . . .
> meanwhile my very best, ingram.
> ps. michael . . . why don't you write a song with the first three words: sauna, sauna, sauna."

The IWMS-2008 was held in Tomar, Portugal (22–26 July 2008) in celebration of the 90th birthday of T. W. Anderson, mentor of George and grand-mentor of Simo, and a long-time Stanford colleague of Ingram's. We invited Ingram as an after-dinner speaker. On 8 April 2008 he wrote:

> "i replied that i didn't want to give an after-dinner talk. i was going over my files and i found the after-dinner talk that i gave in 1998 in florida . . . so what would you think if i gave the same talk . . . maybe with some modest updates. i also kept the photos on transparencies which are different from what simo has."

Unfortunately Ingram was unable to attend the IWMS-2008 in Tomar. On 7 July 2008 he wrote to the IWMS organizers:

> "i think that it may make it easier for everyone if i send you the after-dinner speech that i had in mind. simo is pretty [serious a guy compared with the others][1] so he may be a good choice [to present this after-dinner speech]."

Let us borrow a paragraph from Ingram's after-dinner speech:

> "I was once interviewed and asked who makes the decisions in our family. I knew the answer in a flash—I make all the big decisions, Anita makes all the small decisions. The only problem is that we haven't had a big decision yet in my 63 years of marriage."

[1] wording changed

Section 20.5 in our 2011 *Matrix Tricks* book [12] deals with *How Deviant Can You Be?* — the deviation of any particular observation from the mean, building on Ingram's paper [13, (1992)] and Jensen & Styan [15].
In December 2011 we (Simo & George) had an interesting and pleasant task: we were to prepare a supporting letter to nominate Ingram Olkin for the Hans Schneider Prize in Linear Algebra. For additional support, we contacted Grace Wahba, Professor of Statistics at the University of Wisconsin–Madison, and on 31 December 2011 she wrote us:

> "I wholeheartedly support the proposal that Ingram Olkin be considered for the Prize in Linear Algebra. Absolutely he has to get it!"

Though Ingram did not ultimately receive this particular Prize, on 2 August 2012, he kindly sent us a thank-you e-mail:

> "simo: thanks for your message and in particular i forgot about the award ... however, i am signing George up to write my obituary (assuming he outlives me !!!!!!!) ... I can always count on him. my best, ingram."

## 2 Inequalities: Theory of Majorization

In our supporting letter for the Hans Schneider Prize we pointed out that in our view Ingram's most significant contribution in linear algebra was the book *Inequalities: Theory of Majorization and Its Applications,* with Albert W. Marshall, first published in 1979 [7]. We now have the second edition, with Barry Arnold [8], of the highly-praised classic, without which we know that some people never leave home: now these faithful ones must take into account that the second edition has 909 pages (vs. 569) and its shipping weight is 3.2 pounds (vs. 2.2).
At the end of the first edition of *Inequalities: Theory of Majorization and Its Applications* [7] there is a section on "Biographies" with a photograph of Issai Schur (1875–1941) on page 525. This was the first photograph of Schur that we found and George used it, with the permission of the "publisher and the authors" of [7], in his article on "Schur complements and linear statistical models" [17, (1983/1985)]; see also [18, 19].
Fuzhen Zhang wrote us on 11 May 2016:

> "Dating back to 1984, I went to Beijing Normal University as a graduate student. The first math book in English we used as a text was Ingram's (with Marshall), the 1st edition. I learned and benefited so much from the book. The book has become classical, famous and standard as a reference in this area of research. In 2012, I had the privilege of writing a review for the 2nd edition of the book (published in [20])."

Ingram had a number of Chinese connections, among them was Kai-Tai Fang who in [21, p. 16] tells the following, which is a nice example of Ingram's organizational generosity!

> "During my visit to Stanford University (1981–1982), Professor Ingram Olkin organized a small seminar group on 'multivariate multiple comparisons' which met every week. The participants included T. W. Anderson, Mary Ellen Bock, Zhongguo Cheng and me. ... Then in 1985–1986, upon Professor Ingram Olkin's recommendation, I taught two subjects in the Swiss Federal Institute (ETH, Zürich) as a Guest Professor."

George thinks that he first met Ingram at a colloquium in the Department of Mathematical Statistics at Columbia University in the mid-1960s and at that time may well have served Ingram a cup of tea! Ingram then introduced George to "correlation structure", such as when all the correlation coefficients are equal (intraclass correlation) but the variances are not necessarily all equal. This led to George's Ph.D. thesis [23, (1969)]. See also Ingram's paper on "correlations revisited" (with discussion) [22].
George spent the summer of 1970 at Stanford and he believes it was probably there that Ingram introduced him to the seminal paper by Fan & Hoffman [24, (1955)] in which it is proved that for any $n \times n$ matrix $\mathbf{A}$

$$\mathrm{ch}_j(\mathbf{A} + \mathbf{A}^*)/2 \;\leq\; \mathrm{ch}_j^{1/2}(\mathbf{A}\mathbf{A}^*), \qquad j = 1, 2, \ldots, n. \tag{1}$$

Here $\mathrm{ch}_j$ denotes the $j$th largest eigenvalue. See also Marshall & Olkin [7, p. 240, eq. 4]. The inequalilties (1) were then used by Grossman & Styan in their article on Theil's BLUS residuals [25, (1972)]. And last, but not least, George is most grateful to Ingram for supporting George's appointment as Editor of *The IMS Bulletin*, 1987–1992 [26].
In the "Biographies" section (pp. 528–529) of *Inequalities: Theory of Majorization and Its Applications* [7] there are three photographs of Godfrey Harold Hardy (1877–1947), who with John Edensor Littlewood (1885–1977) and George Pólya (1887–1985) wrote the seminal book *Inequalities* [27], first published in 1934. G. H. Hardy is featured in *The Man Who Knew Infinity* [28], a recent British biographical drama film based on the book [30, (1991)] by Robert Kanigel. The film stars Dev Patel as Srinivasa Ramanujan (1887–1920) and Jeremy Irons as G. H. Hardy. In his movie review, Allan Hunter [29] wrote

> "It tells such a good story that it is hard to resist".

Would a movie about Ingram, *The Man Who Knew Inequalities: Theory of Majorization,* similarly make a good story, hard to resist?

## Acknowledgements

Warm thanks go to Kai-Tai Fang, Michael Greenacre, Jeffrey J. Hunter, Peter Šemrl, Evelyn Matheson Styan, Kimmo Vehkalahti, Grace Wahba, and Fuzhen Zhang for their help.
Photographs: Top left $p_{11}$ at IWMS-1990 Tampere (with Jerzy K. Baksalary & Yadolah Dodge, photograph courtesy University of Tampere); top right $p_{12}$ at IWMS-1995 Montréal (with Gene H. Golub and T. W. Anderson, photograph by Simo Puntanen); bottom left $p_{21}$ at IWMS-1995 Montréal (photograph by Simo Puntanen); bottom right $p_{22}$ at IWMS-2011 Tartu, Estonia (photograph by Jeffrey J. Hunter).

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